Banking and Credit Market Competition with AI and Cryptocurrencies*

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July 12, 2022

Abstract

We study the impact of AI and cryptocurrencies on consumer surplus in banking, on the price of credit, and on the price of checking accounts. We solve a competition model of banking and credit which includes client naivety, heterogeneous client risk, and imperfect risk screening. These features, together, can explain the international pattern of banking costs. In countries where free accounts are prevalent (eg US/UK) both better AI and more crypto use lower consumer surplus, while reducing the amount of naivety improves consumer surplus. Where free-banking is not prevalent (e.g. France/Germany), only one of these three results holds.

Keywords: Overdrafts, Customer naivety, Retail Banking, AI, Cryptocurrencies
JEL Classification: G10, G21, G40

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*We are grateful for insightful and helpful comments to Christine Parlour, two anonymous reviewers, Maryam Farboodi, Zhiguo He, Kebin Ma, Jean Charles Rochet, Ernst-Ludwig von Thadden and seminar participants at the European Economic Association 2020, KU Leuven, and the Network for Industrial Economists 2022. Any errors remain our own.

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1 Introduction

In this paper we study the implications of Artificial Intelligence and cryptocurrencies on consumer surplus in banking, on the price of credit, and on the price of current (or checking) accounts.

There is one prominent empirical fact in banking which, if not explained, brings any predictions on how technology will affect pricing into doubt. This is that, in some countries, regular banking services – personal current or checking accounts which we refer to as PCA – are free for clients in credit. Such accounts are ubiquitous in the UK, widespread in the US, but virtually unknown in France, Italy, Spain, Germany, Belgium and the Netherlands.[1] This paper will offer an answer as to why the practice which has become known as free banking persists in some markets rather than others.[2] And our insights will allow us to make predictions as to how AI and cryptocurrencies will affect the price of banking and credit.

There are perhaps two existing leading theories to explain why free banking sometimes persists. The first builds on differences in client naivety and sophistication. The second theory builds on differences in clients' risk profiles. We will show that neither theory on its own is sufficient to explain the international pattern of bank pricing. And only by considering both naivety and adverse selection due to risk can the differing consumer surplus implications of AI and crypto we discover be demonstrated.

Our main methodological contribution is to combine both of these effects into a richer competitive model of banking and credit. In our Hotelling model of banking competition, some customers are naive, while customers also differ in their riskiness, and the banking industry has an imperfect screening technology which governs the inferences banks can make about their clients. By interacting all of these effects we generate a compelling explanation for the dispersion in prices and profits which we see across countries.

The most recent rationalisation of banking prices rests on heterogeneous naivety amongst clients and is credited to Gabaix and Laibson (2006). This literature argues that some naive customers amongst clients can lead to excessive aftermarket prices, here

[1] We have constructed a data set of the leading current account from the five largest retail banks from each of the six largest Eurozone countries as well as the UK and the US. Annual fees for a standard European account average about €23 (Netherlands) to €100 (Germany).

[2] Free banking has been used widely to denote fee-free banking in the UK and abroad. See for example Goode and Moutinho (1995) or UK Treasury 2014 press release, *New basic fee-free bank accounts to help millions manage their money*. In this paper free banking is unrelated to the historical meaning sometimes intended referring to the era in which banks were free to print their own money, as was the case in the nineteenth century in the United States.

the market for credit, and below-marginal-cost base prices as firms compete to win incumbent clients. However, in the context of banking, explaining prices via naivety faces two challenges: one theoretical, the second empirical.

The theoretical issue is that Gabaix and Laibson (2006), and its successors, do not model competition between the firms in the market for credit.[3] But we note that banks seek to flip customers from their rivals by selling them credit without a current account. In all the developed countries cited the market for credit is huge and competition is strong. If banks can profit both from customers who have accounts with them, and those that do not, then there is no need for such intense competition in the market for current accounts. The link between naivety and free banking is therefore brought into question.

Our work will close this theoretical gap and allow for competition in the market for credit. Doing so maintains naivety as a theoretical explanation for the pattern of banking prices. But there remains an empirical problem with the naivety story. The cross-country pattern of free banking is explained if financial naivety is more prevalent in the UK and US than elsewhere. But there is not support for this empirical proposition. For example, Klapper and Lusardi (2020) report evidence that the US, UK and Germany have the *highest* rates of financial literacy in the world, ahead of those of Belgium and France.[4] While Lusardi and Mitchell (2014) conclude that "*levels of financial literacy found in the United States are also prevalent elsewhere*".

The second approach to explain banking prices is to posit a natural extension of Von Thadden (2004). Von Thadden (2004) argued that if there is heterogeneity in client risk then the bank with the incumbent clients will earn higher profits while prices (for credit) would be dispersed. Although not done, it follows that modelling a first PCA competition stage would result in reduced PCA prices as banks compete for incumbency. But Von Thadden (2004), and the models which build on it, also predict that the challenger bank makes an expected loss from all the high risk clients that she wins. This is in contradiction to the large and profitable business of payday loans, autoloans and other lending products targeting those with more precarious finances.

Combining naivety, heterogeneous risk, and a screening technology in a model raises a number of technical issues as the forces interact in empirically interesting ways. The

[3] Sophisticated customers can instead leave the market and seek the after-market product from firms who are not active in the primary market.

[4] Further, Arrondel, Debbich, and Savignac (2013) report that on standardised tests answered by French respondents, 48% answer the questions on interest correctly, while in the US the comparative figure is 65%. (Lusardi and Mitchell (2014), Table 2.) The results are more balanced on inflation questions.

presence of naivety is the assumption which maintains positive profits even for the rival bank. Heterogeneous risk creates a winner's curse problem for the rival bank and so alters the profit which can be captured from the credit market. In turn the screening technology, which captures AI and crypto, alters the extent of this winner's curse and how it measures up against naivety. And PCA price competition returns some, but not all, of these profits to consumers.

The empirical assumptions which underpin our model are that some clients are naive in all jurisdictions, while credit repayment risk is most pronounced in the free-banking countries: e.g. UK and US. That financial literacy is far from perfect in all jurisdictions is widely supported as noted above.[5] That UK/US might have high proportions of borrowers who pose a significant credit risk is also, in our view, consistent with the available empirical evidence. Clients in the UK and US suffer from more insecure employment: the OECD ranks the US the lowest (least protective) and the UK one of the lowest for the protection of workers in 2019, whilst France and Germany score amongst the highest of all OECD countries.[6] Secondly high levels of debt-fuelled consumption can create high credit-risk borrowers; German households are reported to save 10% of their disposable income, twice as much as the average American, whereas in the UK the savings rate is negative.[7]

The first positive (as opposed to normative) contribution of our work is that our theory of banking and credit, combining naivety and risk, can rationalise observed prices. We then use our model to explore AI and crypto.

The next suite of results we establish concern the consumer surplus implications of better AI and more crypto in free banking countries (such as the UK/US). One might imagine that as AI becomes widespread, consumer surplus would increase as the banking system can establish more accurate information as to their clients. Otherwise, one might suspect that as cryptocurrencies gain dominance, allowing privacy to be better respected, borrowers escape high prices and so consumer surplus can be driven up. We show that neither of these statements is true. If Artificial Intelligence is widespread we show that more AI lowers consumer surplus. Below we explain why greater AI use deepens the winner's curse which raises credit prices leading to this effect. While if crypto use is widespread, even more crypto use lowers consumer surplus again. This time the reason

[5] See Klapper and Lusardi (2020), Lusardi and Mitchell (2014) and references therein.

[6] See Economic Co-operation, Development, Economic Co-operation, and Staff (2020), especially Figure 3.10. The job protection scores are (higher is stronger protection), US 1.31, UK 1.9, Germany 2.33, France 2.68, Belgium 2.71.

[7] See *Why are Germans so obsessed with saving money?*, Financial Times, 22 March 2018.

is that the crypto use alters the adverse selection problem faced by all the banks, raising industry costs and so the price for credit.

Our work allows us to ask the same set of questions for non-free banking countries (such as Germany/France). The answers we find differ for the case of AI, but not for crypto. If crypto use is widespread, even more crypto use lowers consumer surplus. But if AI is widespread then increasing AI use does not lower consumer surplus, nor raise it; consumer surplus is left unaffected. In the dominant AI case, changes in the AI technology result in credit price changes whose effects are returned to clients in the form of cheaper PCA prices. But this profit return effect does not apply when crypto is dominant.

In terms we are able to demonstrate that more AI and more cryptocurrency can both damage consumer surplus. And yet the former improves information and the second harms it. We will explain that the regime change comes about due to the balance between the strength of the naivety versus the adverse selection problem. It follows that these results appear only through the interaction of naivety and adverse selection. Studying each alone would fail to deliver these results.

A further insight of our work is to demonstrate that competition does not return all incumbency profits to clients in the first period, even if there is room to drop prices. Only the second period profit increment between the incumbent and the outsider is competed away in the PCA market in countries with paid banking. This leads to new empirical predictions, explored below, as to the prevalence of free banking as AI levels and crypto use change.

The final suite of results we offer concern the consumer surplus effects of the widespread desire amongst regulators to improve client financial literacy. A case in point is the new UK rules on clear communication to clients as to the price for credit which is linked to current accounts.[8] If every single consumer could be made sophisticated then consumer surplus is maximised in all jurisdictions. But away from this first best, more sophisticated clients always improves consumer surplus in free-banking countries (e.g. US/UK), but can lower consumer surplus in paid-banking countries (e.g. France/Germany).

The dominant effect of reducing naivety in the population is to alter the strength of price competition: incumbent banks tend to price lower, and so an optimal response requires the outsider to price lower too. Overall therefore the price of credit declines – though some distributional nuances between client types remain. In free banking countries

[8]See the Financial Conduct Authority policy statement PS19/25 which requires overdraft pricing (unarranged loans linked to a PCA) to be clearly communicated alongside PCA pricing information. Policy statement available at https://www.fca.org.uk/publication/policy/ps19-25.pdf

this analysis delivers that more sophisticates is good for consumer surplus. In paid banking countries however there is a second-round effect on the first period price for PCAs. This follows as the naivety proportion changes the incremental profits between the outsider and insider. To the extent that more sophisticates reduces the increment in profits between being an outsider and an incumbent, the firms compete less hard in the PCA market, and so these prices rise. On balance therefore consumers can lose out.

The rest of the paper is structured as follows. The next section reviews the relevant literature. Then in Section 3 we introduce our model in its most complete form. This will be a Hotelling competition model in which banks first compete for PCA customers, and subsequently compete in a credit market. Section 4 derives equilibrium with customer naivety and second period competition but without heterogeneity in client risk; it is expositionally easier to extend Gabaix and Laibson (2006) first to allow for competition in the aftermarket. Section 5 then derives equilibrium with risk differences and so adverse selection, and imperfect inference, as well as customer naivety. Section 6 develops our results and discusses their policy implications. Section 7 concludes.

2 Related literature.

There has recently been a renewed focus on understanding competition in banking because of the advent of AI, cryptocurrencies and fintech. This recent literature addresses the role of information in the competition between banks, as we do. He, Huang, and Zhou (2020) focus on competition between a bank which has incumbency and a fintech firm which does not. The two competitors differ in their screening ability, and the paper studies how Open Banking, i.e. customers' right to share their payment history with third parties, may flip the screening advantage from traditional banks to Fintechs. Parlour, Rajan, and Zhu (2020) focuses instead on the first stage: competition between banks and fintech providers to provide banking payment services. This work does not address the subsequent market for credit as a monopoly supplier of credit is assumed. Our work contributes to this literature by modelling competition in both the market for credit and the prior market for current accounts, that is the market for incumbency. Finally, Ahnert, Hoffmann, and Monet (2022) studies the complementary question of the optimal means of payment - cash, bank deposits, CBDC or digital tokens - by merchants. In contrast to ours, their main focus is to illustrate how privacy aspects of transacting can induce socially efficient trading decisions.

This new literature typically studies risk heterogeneity as the only client friction. We go further and study risk and naivety.

Naivety is arguably the dominant explanation for the prevalence of free-banking and is credited to Gabaix and Laibson (2006). In a typical equilibrium of this modelling tradition, applied to banking, banks either have a monopoly on the aftermarket service – assuming away credit markets – or clients can substitute an out-of-market solution and so have an outside option utility. In either case naive consumers pay high after-market prices and so are valuable. In turn the banks compete to win these clients, pushing PCA prices down.[9] This 'shrouding equilibrium' has been identified with free banking in the banking literature (e.g. Armstrong and Vickers (2012), Heidhues, Kőszegi, and Murooka (2016a), Heidhues, Kőszegi, and Murooka (2016b)). If sophisticated customers seek alternative sources of credit, e.g. credit cards, then they are ultimately receiving credit from the banking system and so from rival banks. One of our contributions is to allow for this profit from poaching rival's clients.

We noted in the Introduction some of the empirical evidence for naivety throughout the world in the form of financial literacy scores. Alan, Cemalcilar, Karlan, and Zinman (2018) provides direct evidence of the exploitation of naive customers in Turkey using an overdraft market experiment: by randomizing messages which affect consumers' attention in various ways they demonstrate unawareness of prices and underestimation of future usage.[10]

The adverse selection component in our model builds on Von Thadden (2004), who in turn built on Sharpe (1990) and Rajan (1992). Von Thadden (2004) is the first paper to solve for the equilibrium dispersion in the price of credit under competition. He, Huang, and Zhou (2020) use a similar methodology, as do we. Our contribution is to allow for consumer naivety as well as heterogeneous risk. Naivety preserves positive profits across market participants and moderates the winner's curse effect and adverse selection. In turn the model affords a richer explanation of cross-country differences.

That modern technology (e.g. cashless payments) is impacting the ability of credit market competitors to alter their prices and approval decisions is evidenced by Ghosh, Vallee, and Zeng (2021). Our work on the consumer surplus implications of naivety, AI

[9]The early literature on aftermarket pricing (Shapiro (1994)) recognizes that monopoly profits are distributed back to the customers if prices can be decreased sufficiently on a competitive base-good market. However, a lower bound on the price can prevent competing away these profits (Heidhues, Kőszegi, and Murooka (2016a)) and so will be the source of economic rent. This lower bound can arise endogenously in certain markets (Miao (2010)).

[10]Other notable empirical studies on overdraft fees and customer naivety are Adams (2017), Stango and Zinman (2009), Stango and Zinman (2014), Morgan, Strain, and Seblani (2012), Melzer and Morgan (2015),Williams (2016).

and crypto builds on this insight and is related to recent research on the surplus implications of price discrimination based on customer naivety in the context of financial services. Kosfeld and Schüwer (2017) shows that educating customers may have unintended welfare consequences if naivety-based discrimination is possible in the aftermarket. Heidhues and Kőszegi (2017) focus on the welfare aspects in more general settings, and provide conditions where naivety-based discrimination negatively affects welfare. Our screening technology extends these works and allows us to discuss AI and cryptocurrencies as well as naivety.

3 Model setup

Consider the following two-stage game of competition in a retail banking market which takes place at times $t = 1$ and $t = 2$. Two banks ($j \in \{A, B\}$) offer a personal current or checking account (PCA) in period one, and a borrowing facility[11] in period two. A unit measure of customers with a fixed demand for exactly one PCA in period 1 are located uniformly over the interval $[0, 1]$, while the two banks are located at the two opposite ends of the interval. In the first period ($t = 1$), a customer located at $\gamma \in [0, 1]$ incurs a transportation cost $\tau \cdot \gamma$ to obtain their PCA from bank A, and $\tau \cdot (1 - \gamma)$ to obtain their PCA from bank B, where τ is the Hotelling transport cost.

In the second period ($t = 2$) the customers may need access to credit (e.g. overdrafts, credit cards, auto loans). In this credit market stage, products are homogeneous, and there is no transportation cost.[12] This specification reflects the idea that factors such as product differentiation, branding, or even the physical location of branches are more pronounced when choosing one's primary provider of banking services, while the borrowing facility – the lending of money to cover a liquidity need – is homogeneous across the banks.

At $t = 1$ each bank $j \in \{A, B\}$ simultaneously announces a fee $p_j \geq 0$ for the PCA, which is observable to all customers.[13] Following the price announcement, customers

[11]This borrowing facility can represent an overdraft service, when offered by one's own relationship bank, or a credit card, payday lending, e-money, or other arms-length transaction financing when offered by a 3rd party.

[12]Gill and Thanassoulis (2009), and Gill and Thanassoulis (2016) use a similar approach of strategic competition in a first stage followed by Bertrand competition in a second stage. In these papers the follow on study concerned the ability of consumers to bargain with sellers over the list price.

[13]The price floor in stage 1 could be altered without loss of generality. The change in the analysis would be immediate. Banks can offer slightly negative prices through gifts. However these negative prices are not substantial as evidenced by the low PCA customer switching rates in many countries (e.g.

choose exactly one bank, which we then refer to as their *insider bank*. In period $t = 2$ a fraction η of customers are hit by a liquidity shock, and need to use a borrowing facility. In this period they can decide whether to use the facility offered by their own bank (overdraft or credit card), or by the competitor bank (credit card, payday loan, auto loan); we refer to the two actions as "STAY" and "SWITCH".

Customers differ in two aspects: (i) sophistication and (ii) riskiness. Sophisticated customers (type S, fraction $1 - \alpha$) are fully rational, anticipate the probability with which they will need credit, and in both stages choose the bank with lower expected total outlay, including any transportation costs. If the expected payment is equal, they choose randomly in the first stage, and use the insider bank's facility in the second stage. Naive customers (type N, fraction α) fail to predict their future demand for the borrowing facility, therefore in period 1 they base their decision only on the observable first-period prices. This would be the case, for example, if naive customers were overconfident in their ability to avoid needing an overdraft in the future. Furthermore, in the second period, should they need credit, naive customers do not consider the possibility of using alternative providers and always use the borrowing facility offered by their insider bank. One possible justification of this assumption is that without planning they become involuntary overdraft users.

A proportion β of customers default on a credit contract with higher probability and in turn yield reduced profit to the bank which serves them (type H consumers, denoting high credit-risk), while a fraction $(1 - \beta)$ are low credit-risk (type L consumers), and so more profitable. A customer's expected cost to a bank is captured concisely by parameters $c_L < c_H$, representing exogenous default costs (or alternatively the operational costs of recovering the outlay from the customer).

Each insider bank receives a signal $\{\ell, h\}$ as to the riskiness of each of her clients. This signal captures the inferences which the bank is able to make from the information generated by the operational relationship between the bank and the client (through the PCA) prior to credit being sought. The bank can use this information to price discriminate in the second period.[14]If the borrower is a high-risk type (H), then the insider bank receives a signal h with probability λ. Otherwise the low signal ℓ is received. Low risk clients always generate a signal ℓ.

The signal structure is depicted in Table 1. The signal structure has the feature that

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UK).

[14]For example Melzer and Morgan (2015) documents empirical evidence that banks take into consideration the credit risk of borrowers when granting overdraft facilities for customers, and they strategically react to outside competition.

		Borrower risk	
		L	H
Pr signal received:	ℓ	1	$1-\lambda$
	h	0	λ

Table 1: Signal structure for insider bank

a signal of h is fully informative – the client is high risk. In common with other papers in the literature we therefore have a bad news structure.[15] This captures that if there is evidence of high risk, such as prior default or erratic deposits, then the insider bank can be certain that the client is high risk. However absence of evidence of riskiness is not evidence of absence. Thus a signal of ℓ is only partially informative.

The quality of the signal is captured by the parameter λ. If $\lambda = 1$ then the signal is perfectly informative. Each bank is assumed to have the same signal quality generated by their PCA customers, so we are modelling the industry-wide technology available. This allows us to parsimoniously capture some implications of cryptocurrencies and of AI in banking, as we now explain.

Recourse by customers to the use of crypto currencies hides transactions from the bank. Erratic cashflows can be hidden from a bank by receiving bitcoins and the like. Hence a greater prevalence of crypto use in the population would imply a reduced λ – the quality of the banks' signals would diminish. Alternatively, improved use of artificial intelligence can allow the clues as to a client's riskiness to be unearthed from the confidential banking relationship data with greater certainty. Hence improved AI would imply a higher λ – the information advantage available to the insider bank would increase.[16]

We assume that all dimensions of heterogeneity (naivety, riskiness, exposure to liquidity shocks, signal, and location of the customer) are independently distributed in the population. The joint probability mass function of a customer is completely characterized by parameters α, β, η and λ, which are assumed to be exogenous throughout the analysis.

At $t = 2$, as noted above, bank $j \in \{A, B\}$ can condition the interest rate (fee) for its insider customers on its stage 1 market share and on its clients' riskiness *signal*, but not on sophistication *per se*. Therefore, the borrowing facility (overdraft) is offered at

[15]He, Huang, and Zhou, 2020 for example also use a bad news structure.

[16]Improved AI would allow all data owners to make better inferences from their data. At present data privacy is an increasing focus of regulation and so the main improvement in inference will be made by the insider bank which has access to its relationship banking data.

a price $r_h^{(j)}$ and $r_\ell^{(j)}$ for customers depending on the signal received by the insider bank $j \in \{A, B\}$. Furthermore, Bank j offers a borrowing facility for customers of the other bank *(outsider customers)* at an on-demand price $r_o^{(j)}$. The price for consumer credit, both provided by the insider and the outsider bank, is assumed exogenously capped[17] at a common value $\overline{r}$ satisfying $c_H < \overline{r}$. The marginal cost of opening and maintaining a PCA is normalized to 0, while all operating costs related to the credit facility are absorbed by parameters c_L and c_H.

Note that riskiness of cashflow is observable to the insider bank and so a signal is generated. The banks do not however observe client naivety. This is standard in the two traditions of heterogeneous risk and naivety (Von Thadden, 2004, Gabaix and Laibson, 2006). We believe this is a good assumption as naivety encompasses an unwillingness to search (e.g. because the client thinks their time is too valuable), as well as clients who do not expect to suffer a liquidity shock (e.g. because they fail to take sufficient account of their job security or the possibility of accidents), besides the often thought of clients with an inability to search (e.g. due to poor education or financial illiteracy). Thus consumers who would behave as if they were naive can be present amongst those with secure cashflows into their account, or those who choose to sometimes use cryptocurrencies, as well as clients with erratic incomings.

We will assume that customers' valuations of the account and the credit service are sufficiently high so that all customers decide to consume in all equilibria which we focus on in the main text. This assumption of full coverage is common in competition models.

We solve the game for Perfect Bayesian Equilibrium. Let $a_1 \in \{A, B\}$ and $a_2 \in \{\text{“STAY”}, \text{“SWITCH”}\}$ denote customers' decisions in the first and the second period respectively. Then:

Definition 1 *A Perfect Bayesian Equilibrium of the PCA-pricing game consists of*

1. *A first-period PCA price by the two banks:* $\{p_A^\star; p_B^\star\}$*;*
2. *Customers' decision over which bank to choose in the first stage* $a_1^\star$*;*
3. *Second-period credit fees conditioned on signals by the insider:* $\left\{r_\ell^{A\star}, r_h^{A\star}, r_o^{A\star}; r_\ell^{B\star}, r_h^{B\star}, r_o^{B\star}\right\}$;
4. *Customers' decision whether to use their own bank's service (‘STAY’) or the competitor's service (‘SWITCH’):* $a_2^\star$

[17]For example, the UK has introduced a price cap on short-term high-cost credit in 2015 (see Financial Conduct Authority (UK), PS14/16 Policy Statement: Detailed rules for the price cap on high-cost short-term credit, November, 2014. Policy statement available at https://www.fca.org.uk/publication/policy/ps14-16.pdf.)

where decisions are sequentially rational:
(i) each bank maximizes profit at each stage given anticipated customer's behaviour, and
(ii) sophisticated customers' decisions a_1 and a_2 are optimal given their belief regarding equilibrium prices and their need or otherwise of t=2 liquidity; naive customers' decision a_1 is boundedly rational given their belief that they will not be subject to a liquidity shock.

Notice that the timing of the model induces a 4-stage strategic-form game, but it is useful to consider it, as described above, as a game which unfolds in two periods: at $t = 1$ banks offer current accounts and customers engage with exactly one of them, while at $t = 2$ banks offer an add-on liquidity service and affected customers decide whether to use their own bank's facility or seek credit from the rival provider. We solve the game backwards: first we determine the $t = 2$ equilibrium, taking prices and customer-bank relationships from $t = 1$ as given. Then, we consider the equilibrium of the overall game.[18]

We finally note that our Hotelling specification is as parsimonious as possible consistent with providing sufficient structure for our results to emerge. If we collapsed the first stage Hotelling and assumed that banks were homogeneous in current (or checking) accounts then the model would generate Bertrand pricing in many settings which make it difficult to explain countries with above marginal cost pricing (paid-banking as in Europe). Alternatively, adding taste parameters into the credit competition stage would not simplify but rather complicate as there would remain the need for mixed strategy equilibria due to naivety.[19]

4 Credit markets and customer naivety

This section establishes how the presence of some naive customers alters competitive outcomes in the credit market, and therefore in the banking PCA market, setting aside adverse selection concerns. Allowing rivals to win business off each other in the secondary (add-on) market offers our first extension to the existing literature.

This section therefore studies the special case where $\beta = 0$; all consumers are low credit risk type. In this case there is no information asymmetry between the insider and the outsider bank, and so there is no adverse selection problem. As there is only one

[18] The Perfect Bayesian Equilibrium concept is facilitated in this game as we are able to avoid off-equilibrium belief concerns. The second round takes any initial outcome and so there are no off-equilibrium issues. In the first round, as there are a continuum of borrowers, no individual can affect global parameters and so off-equilibrium beliefs do not come into play.

[19] These mixed strategy would be more complicated as the upper bound on prices would vary in the population as the distance to the firm varied.

customer type, we drop riskiness indices to simplify notation: let $\{r^{(j)}, r_o^{(j)}\}$ denote the price of credit offered by bank $j \in \{A, B\}$ for insider and outsider customers respectively, and c denote the net marginal cost of providing credit.

4.1 Second-period equilibrium

Suppose that bank $j \in \{A, B\}$ starts with a mass l_j of customers who are subject to probabilistic liquidity shocks, and within these customers, the percentage of naive types is α_j. Notice that the subscripts allow for the possibility that naive and sophisticated consumers make different choices at $t = 1$. The second-period subgame is separable into two distinct components: banks compete for the insider customers of Bank j through the choice of $r^{(j)}$ and $r_o^{(j)}$, for each $j \in \{A, B\}$. Given this observation, we can formulate the subgame from an arbitrary bank's perspective as follows: (i) two banks jointly announce a credit fee $\{r^{(j)}, r_o^{(-j)}\}$, and (ii) all sophisticated customers of j who need liquidity, that is, $\eta l_j(1 - \alpha_j)$ mass of customers, use the outside facility if and only if $r_o^{(j)} < r^{(j)}$.

This subgame has no Pure-strategy Nash equilibrium. To understand this intuitively, consider that the insider bank can always unilaterally deviate to the strategy of serving naive customers only at the maximum fee $\overline{r}$, and so obtain a positive economic profit. This in turn implies that the insider bank's lowest possible offer in any pure-strategy equilibrium will be bounded away from the marginal cost c, and will be undercut by the competitor bank in a Bertrand competition. Consequently, the best response correspondence has no fixed point. Proposition 1 formally proves this and establishes the equilibrium in mixed strategies for the second-stage credit market subgame.

Proposition 1 *If $\alpha_j > 0$ and $\beta = 0$ (all consumers are low-risk type) the 2nd-period credit pricing game has no Pure-strategy Nash equilibrium. The unique mixed-strategy Nash equilibrium (MSNE) is as follows: both insider bank $j \in \{A, B\}$ and outsider bank $(-j)$ mix over the interval $[\underline{r}_j, \overline{r}]$ where $\underline{r}_j$ is:*

$$\underline{r}_j = \alpha_j \overline{r} + (1 - \alpha_j)c. \tag{1}$$

In the unique MSNE the outsider bank mixes according to continuous distribution F_{out}, *and the insider bank mixes according to F_{in} as defined below:*

$$F_{in}^{(j)}(r) = 1 - \alpha_j \frac{\overline{r} - c}{r - c} \tag{2}$$

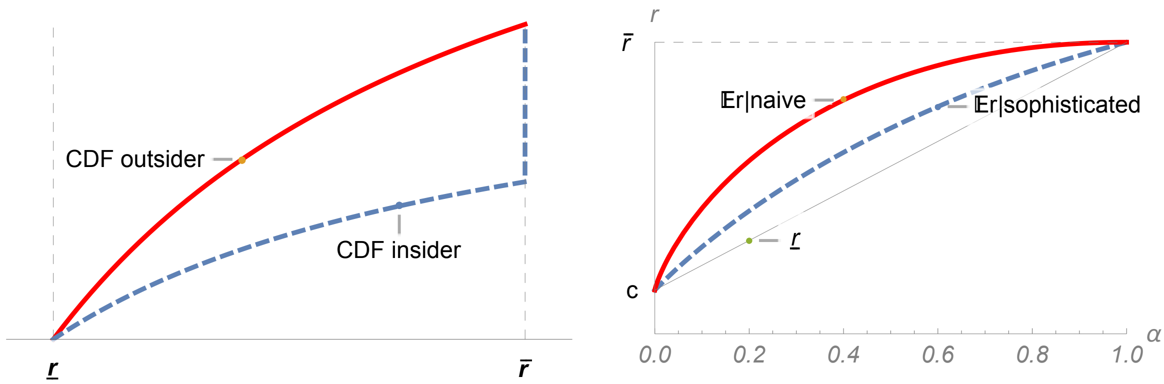
$$F_{out}^{(-j)}(r) = \frac{1}{1-\alpha_j} - \frac{\alpha_j}{1-\alpha_j} \frac{\overline{r} - c}{r - c} \tag{3}$$

The insider bank places a probability mass of α_j on $\overline{r}$.

Proof. *See Appendix A.1* ■

Figure 1: Second period equilibrium
Customer naivety without adverse selection

(a) Credit fee dispersion (b) Expected credit fee



Panel (a) illustrates credit fee dispersion offered by the insider bank (blue, dashed) and the outsider bank (red, solid) for $\alpha = 0.5$. Panel (b) depicts expected credit fees paid by sophisticated (blue,dashed) and naive (red, solid) customers as a function of α, the proportion of naive customers.

Intuitively, the equilibrium credit fee dispersion (Figure 1, panel (a)) captures the following trade-off: an insider bank strikes a balance between exploiting its naive customers (by setting a high credit fee) and competing to keep its sophisticated customers (by decreasing the fee). At one extreme, when all consumers of bank j are naive ($\alpha_j \to 1$), the CDF converges to a single mass-point at $\overline{r}$ (maximal exploitation), while at the other extreme of all sophisticated consumers ($\alpha_j \to 0$) the CDF converges to a mass-point at $\underline{r}$, which itself converges to marginal cost c (competitive outcome).

It is the presence of naive customers which initiates price dispersion in the credit market stage. As the proportion of naive consumers of bank j (α_j) increases, the insider places more-and-more probability mass on higher fees. This allows the outsider bank to also increase its fees probabilistically. The presence of naive customers deters banks from competing fiercely for the sophisticated customers, and so sophisticated consumers end up paying a markup over marginal cost. We quantify this markup in Lemma 1.

Lemma 1 *The expected credit fee paid by sophisticated and naive customers of bank* $j \in \{A, B\}$ *is:*

$$\mathbb{E}r_j^{soph} = c + \left(2\alpha_j + \frac{\alpha_j^2 \ln[\alpha_j]}{1-\alpha_j}\right)(\overline{r} - c)$$
$$\mathbb{E}r_j^{naive} = c + \alpha_j (1 - \ln[\alpha_j])(\overline{r} - c).$$

The probability that the outsider wins the sophisticated customers is a linear function of the mass of naive customers (α_j)*:*

$$Prob[r_o^{(-j)} < r^{(j)}] = \frac{1}{2} + \frac{\alpha_j}{2}.$$

Proof. *See Appendix A.2* ■

Lemma 1 uses the pricing distributions determined in Proposition 1 to establish the extent to which the expected fee paid by both naive and sophisticated consumers rises above the competitive level (c) as a result of the presence of naive consumers. The expected credit price is found from the minimum of the probabilistic credit price offered by the insider and outsider banks. These expected credit prices can be depicted graphically as a function of the proportion of naive consumers in the population (α), and this is done in Figure 1, panel (b).

As $\alpha_j \to 0$, there is no naive distortion effect, credit fees converge to the competitive outcome $r = r_o = c$ and the outsider wins with a probability of 1/2. As the mass of naive customers increases, both price distributions move to higher prices in a first order stochastically dominant manner. This increases the expected cost of credit for sophisticated customers as well as the probability that the outsider bank wins the price competition. As $\alpha_j \to 1$ the probability mass on $\overline{r}$ goes to 1, and the outsider wins the few remaining sophisticated customers with probability 1.

Before turning to the first stage, we must analyse the profits obtained by banks from the second period credit market. From the indifference conditions established as part of the proof of Proposition 1 we know that bank j's expected profit from its role as an insider and as an outsider is:

$$\pi_{in}^{(j)} = \eta l_j \alpha_j \left(\overline{r} - c\right) \quad \text{from (A.7)}$$
$$\pi_{out}^{(j)} = \eta l_{-j} (1 - \alpha_{-j}) \left(\underline{r}_{-j} - c\right) \quad \text{from (A.4)}$$

The outsider profit comes entirely from sophisticated customers of the rival bank who switch, while the insider profit consists of two components: sophisticated customers who decide to stay optimally, and naive customers who always stay by assumption. The following Lemma derives banks' aggregate profit.

Lemma 2 *For arbitrary values of market share l_j and proportion of naive consumers amongst clients α_j, bank j's profit from the 2nd period credit market is:*

$$\pi^{(j)} = \left[\alpha - \frac{\left(\alpha - \alpha_j l_j\right)^2}{1 - l_j}\right] \eta \left(\overline{r} - c\right) \tag{4}$$

with bank index $j \in \{A, B\}$.

Proof. See Appendix A.3 ■

4.2 First-period equilibrium

We now move to competition in the first period and solve the game.

Recall that naive consumers do not believe they will require credit. Such consumers choose a bank taking into account the first-period price vector $\{p_A, p_B\}$ only. Therefore a naive customer selects Bank A if and only if she is located at $\gamma < \gamma^N$, where γ^N is determined by the standard Hotelling indifference condition:

$$p_A + \tau \gamma^N = p_B + \tau (1 - \gamma^N).$$

The value of this threshold is:

$$\hat{\gamma}^N = \frac{1}{2} + \frac{p_B - p_A}{2\tau}. \tag{5}$$

Sophisticated customers correctly anticipate that they will need credit with some probability, and, being rational, predict the $t = 2$ equilibrium price and calculate their total expected payments to each bank. Sophisticated buyers' choices exert an externality on each other: the more such buyers who choose bank A say, the lower that bank's proportion of naive consumers α_A.

Lemma 3 *For any PCA price vector $\{p_A, p_B\}$ such that $\hat{\gamma}^N(p_A, p_B) \in (0,1)$ there exist an equilibrium of the induced subgame where (i) sophisticated and naive customers follow a common threshold defined in Equation* (5) *— now denoted as $\hat{\gamma}$ — so that only customers with $\gamma_i < \hat{\gamma}$ choose Bank A, and (ii) second-period credit fees are determined according to Proposition 1, with $\alpha_A = \alpha_B = \alpha$.*
Proof. *See Appendix A.4* ■

Lemma 3 is a natural consequence of the fact that the second period pricing strategies (Proposition 1) are a function of the proportion of naive consumers at each bank, and not on each firm's market shares. Therefore suppose the market is in equilibrium with sophisticated customers mimicking the period 1 behaviour of unsophisticated customers. It follows that each bank secures a representative sample of borrowers, and so the proportion of sophisticated customers at each bank matches the population proportion α. The second period credit pricing strategies will therefore be symmetric across the two banks. So second period pricing will not create an incentive for sophisticates to alter their decisions, confirming the equilibrium.

It follows from Lemma 3 that decreasing the $t = 1$ price of PCA increases market share. Bank A's total expected profit, using Lemma 2 with $\alpha_A = \alpha_B = \alpha$ (see (A.10)) is:

$$\Pi^A = \hat{\gamma} p_A + \pi^A = \hat{\gamma}(p_A, p_B) p_A + \left(\alpha(1-\alpha) + \hat{\gamma}(p_A, p_B)\eta\alpha^2\right)(\overline{r} - c) \tag{6}$$

For any given p_B, the optimal choice of p_A is given by the first-order condition:

$$\frac{\partial \Pi^A}{\partial p_A} = \frac{\partial \hat{\gamma}}{\partial p_A} p_A + \hat{\gamma} + \eta\alpha^2 (\overline{r} - c) \frac{\partial \hat{\gamma}}{\partial p_A} = 0$$

Note that $\frac{\partial \hat{\gamma}}{\partial p_A} = -\frac{1}{2\tau}$ therefore bank A's best response function is:

$$p_A = \frac{1}{2}\left(p_B + \tau - \eta\alpha^2 (\overline{r} - c)\right)$$

Solving for symmetric equilibrium we derive:

$$p_A = p_B = p = \tau - \eta\alpha^2 (\overline{r} - c) \tag{7}$$

which uniquely characterises the symmetric equilibrium. So summing up, we have shown:

Proposition 2 *The unique symmetric equilibrium of bank competition without adverse selection* ($\beta = 0$) *is as follows. If* $\tau \leq \eta\alpha^2 (\bar{r} - c)$*, then the only symmetric equilibrium is* $p_A = p_B = 0$*. Otherwise there exists a unique symmetric-price equilibrium, with first-period PCA prices defined by*

$$p^\star = \tau - \eta\alpha^2 (\bar{r} - c) . \tag{8}$$

Proof. *Follows from above.* ■

The price of current accounts in this differentiated market is reduced as compared to the standard Hotelling level for the PCA market alone (τ) by an amount which increases more than linearly in the proportion of naive consumers in the population (α). These naive consumers are valuable at time $t = 2$ when customer credit is sold. Below we discuss why.

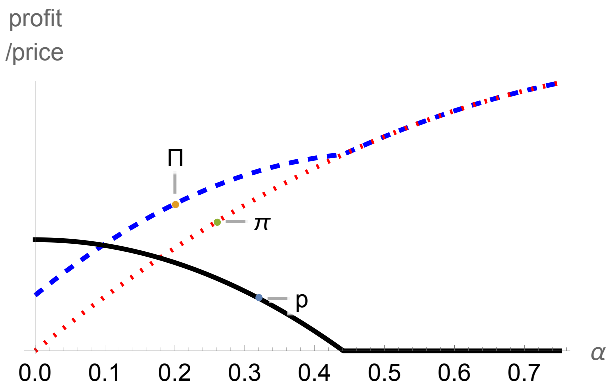
When the competitive constraint in the PCA market is strong enough, $\tau < \eta\alpha^2(\bar{r} - c)$ the best response price of (8) is negative. In this case, within the feasible range of parameters, decreasing first-period price would always be a profitable deviation, and first-period PCA prices hit the lower bound. Rearranging this expression for α, we obtain that free banking prevails whenever the mass of naive customers exceeds a threshold, and this threshold converges to zero as the first-period competition parameter (τ) converges to perfect competition.

4.3 Discussion

We have established the symmetric equilibrium solution to the PCA and credit market when some consumers are naive, but, in keeping with the Gabaix and Laibson (2006) tradition, when adverse selection issues are absent.

Equilibrium profits can be computed using (6) and are graphed in Figure 2. In the second period the more naive consumers a bank has secured as its customers, the less competitive the bank is in its credit pricing as the bank seeks to profit from its clients' naivety. This creates an incentive to win customers in the first period and acts to pull PCA prices down. This is the Gabaix and Laibson (2006) intuition. There is a secondary effect however. As the bank becomes less competitive in the second period, the rival bank can win some of the sophisticated customers at a higher price, raising its profits. This reduces the need to secure high first period market shares as profitable sophisticates can be won from the rival in the second period. This is a new effect and acts to reduce rivalry

Figure 2: Equilibrium profit and PCA price
Customer naivety without adverse selection



Dashed (dotted) red lines denote profits in period 1 (PCA market) and 2 (credit market) respectively. Blue (solid) line denotes aggregate bank profit. Black solid line denotes equilibrium PCA-price.

in the first period, raising first period prices. Combining we see that the combination of these effects, captured in Figure 2, in contrast to the Gabaix and Laibson (2006)-tradition, results in strictly positive profit emerging for any mass of naive customers.

Proposition 2 delivers the result that greater naivety amongst the banking population leads to lower bank PCA prices. As we noted therefore, the international pattern of the costs to consumers of banking services could be explained if countries like the UK and US have higher levels of unsophisticated clients than France and Germany for example. However, we noted that this assumption does not reflect the available evidence.

5 Credit markets with naivety and adverse selection

So far the analysis has set adverse selection aside. This is problematic as the strategic effects of competition in credit markets are critical to equilibrium PCA prices, and yet adverse selection is a natural feature in such competition. We therefore now turn to the full model and analyse the interaction between customer naivety and adverse selection. This will develop our main methodological contribution to the literature.

In this section we demonstrate that equilibrium in the market takes the following form: the insider bank sets a price for borrowers with an h-signal and borrowers with an ℓ-signal according to independent distributions F^h_{in} and F^ℓ_{in}, while the outsider bank, which cannot distinguish high versus low risk customers, sets credit prices according to the distribution F_{out}. The model is solved backwards starting with the equilibrium for the second period, then iterating back to solve the first period game.

5.1 Second-period credit market equilibrium

We establish existence of an equilibrium in mixed strategies in which the insider bank randomizes independently for low types and high types over adjacent intervals $[\underline{r}, \hat{r}]$ and $[\hat{r}, \overline{r}]$ respectively, while the outsider bank randomizes according to a piecewise-continuous distribution over the union of these intervals. So credit prices for customers with an ℓ-signal are drawn by the insider bank from F_{in}^{ℓ} supported on $[\underline{r}, \hat{r}]$. The proof consists of two main steps: first, we show that any equilibrium must satisfy this structure. Then, we derive price dispersion taking the structure as given. In the main text we provide a sketch of the proof focusing on intuition, while rigorous proof and analytical calculations are relegated to Appendix B.

For this section, we introduce the following notation: $\pi(\theta, r, \rho)$ denotes profit from serving a unit measure of customers of type $\theta \in \{\ell, h, \ell h\}$ with credit, when the fee for credit is r, and the bank wins type θ customers with probability ρ. Type ℓh denotes that customers are distributed according to population probabilities. Subscripts $_{in}$ and $_{out}$ refer to profit for the bank from its insider and outsider role, respectively. $\underline{\pi}(\theta)$ stands for a minimax payoff according to a well-defined, feasible alternative strategy.

First, notice that the insider bank can always revert to the strategy of serving naive customers only, and charging the fee cap $\overline{r}$. This defines two candidate "minimax" payoffs for the insider bank, one when serving clients who generate a low signal, and a second for clients who generate an h signal. It will be proven that as both groups of customers are faced with the same outsider price distribution F_{out}, exactly one of these minimax payoffs will be binding.

Suppose first that the minimax payoff from the high risk types, $\underline{\pi}_{in}(h)$ is binding. This profit pins down the upper piece of the outsider bank's piecewise-defined CDF (denoted F_{out}^{h})[20] through the insider's indifference condition for the high-type customers. That is, the insider bank is willing to offer an arbitrary r to high type consumers only if its expected profit from charging r equals its minimax profit:

$$\pi_{in}\left(h, r_h, Pr[r_h \leq r_o]\right) = \underline{\pi}_{in}\left(h\right).$$

Suppose now that we know the cutoff value of the insider bank's fee distribution, $\hat{r}$,

[20] Note that although we use analogous notation, F_{in}^{ℓ} and F_{in}^{h} denote two different distributions, while F_{out}^{ℓ} and F_{out}^{h} is one piecewise-defined continuous CDF.

which is the lower bound of the support of F_{in}^{h} and the upper bound of the support of F_{in}^{ℓ}. Then it would be possible to formulate the profit for the insider bank from serving low risk type customers. By offering $\hat{r}$ to ones low risk clients, the insider bank wins with probability $Pr[\hat{r} \leq r_o]$, which we denote by $\hat{\rho}_o := 1 - F_{out}(\hat{r})$, and, her payoff from serving low type consumers is $\pi_{in}(\ell, \hat{r}, \hat{\rho}_o)$. Due to the insider's indifference property, this must be equal to the expected payoff at the lower boundary of F_{in}^{ℓ}, labelled $\underline{r}$. By playing $\underline{r}$ the insider bank wins with probability 1, so $\underline{r}$ is the value of r which solves

$$\pi_{in}(\ell, \underline{r}, 1) = \pi_{in}(\ell, \hat{r}, \hat{\rho}_o). \tag{9}$$

Notice that (9) defines the common lower boundary of the support of F_{in}^{ℓ} and F_{out} as a function of $\hat{r}$.

If the outsider's price is $\underline{r}(\hat{r})$, the outsider wins the competition with probability 1 and obtains all ℓ and h sophisticated types, generating profit $\pi_o\left(\ell h, \underline{r}(\hat{r}), 1\right)$. Due to the outsider's indifference condition this must be equal to its profit when charging $\overline{r} - \epsilon$ (with $\epsilon \to 0$). In the latter case the outsider wins high-type customers only, with the probability that the insider bank places on $\overline{r}$. Let this probability be $\overline{\rho}_i$, which can therefore also be written as a function of $\hat{r}$, and the associated profit is labeled $\pi_{out}(h, \overline{r}, \overline{\rho}_i(\hat{r}))$.

The next step is to determine the function $\overline{\rho}_i(\hat{r})$, that is, the mass the insider places on the upper bound, $\overline{r}$, as a function of the cutoff, $\hat{r}$. By construction, $\hat{r}$ is the lower bound of F_{in}^{h}. Therefore, the outsider bank's indifference condition at the two boundaries of the support of F_{in}^{h} is

$$\pi_{out}(h, \hat{r}, 1) = \pi_{out}(h, \overline{r}, \overline{\rho}_i(\hat{r}))$$

This defines implicitly the insider's probability mass at $\overline{r}$ as a function of $\hat{r}$, that is, $\overline{\rho}_i(\hat{r})$.

The final step combines the lower and upper part of outsider's indifference condition to solve for the unique threshold value $\hat{r}$. At the equilibrium value of $\hat{r}$ the outsider bank must be indifferent between charging $\underline{r}(\hat{r})$ and winning both types with probability 1, or charging $\overline{r} - \epsilon$ (with $\epsilon \to 0$) and winning the high types with probability $\overline{\rho}_i(\hat{r})$. That is

$$\pi_{out}(\ell h, \underline{r}(\hat{r}), 1) = \pi_{out}(h, \overline{r}, \overline{\rho}_i(\hat{r})). \tag{10}$$

We solve equation (10) in Appendix B to derive analytically the equilibrium value of $\hat{r}$:

$$\hat{r}^{\star} = c_H + \frac{\alpha_j}{\beta\lambda}\left(\overline{r} - c_H\right). \tag{11}$$

The result shows that as $\alpha_j \to \beta\lambda$, $\hat{r}^\star \to \overline{r}$. As a consequence, for every value of $\alpha_j > \beta\lambda$, the case when adverse selection is not the main friction, but naivety is, the fee cap is binding for the high types, leading to an equilibrium where F_{in}^h is degenerate.

For the more complex case of $\alpha_j < \beta\lambda$, next, we use $\hat{r}^\star$ to pin down the insider's equilibrium profit: at $\hat{r}^\star$ the insider wins the low-types with probability $(1 - F_{out}(\hat{r}^\star))$, which defines its modified equilibrium payoff from serving low-types, $\pi_{in}^{\ell\star}$. We show that the equilibrium profit from the low types exceeds the minimax payoff, that is, the payoff from serving naive low-types only, verifying the claim that only the minimax payoff for the high type is binding. As the equilibrium payoff is pinned down, we can write the insider's indifference condition against low-types as:

$$\pi_{in}(\ell, r, Pr[r_\ell \le r_o]) = \pi_{in}^{\ell\star}.$$

This equation defines the functional form for $F_{out}(r)$ on the interval $[\underline{r}, \hat{r}^\star]$.

Finally, the insider bank's price dispersion is derived using the outsider's indifference condition: at every r, the outsider bank must be indifferent between playing r or its alternative payoff, which is pinned down by the mass-point the insider places on $\overline{r}$. This leads to the two independent indifference conditions, generating the two distributions we are after. Proposition 3 fully characterizes the second period equilibrium credit fee dispersion.

Proposition 3 *The equilibrium of the second period credit pricing game with naivety and adverse selection is as follows. The insider bank j mixes over $[\underline{r}, \hat{r}]$ according to $F_{in}^\ell(r)$ for customers with an ℓ-signal, and over $[\hat{r}, \overline{r}]$ according to $F_{in}^h(r)$ for customers with a h-signal, with $\hat{r}$ defined in (11). The insider places a positive mass $\overline{\rho}_i$ on $\overline{r}$ where*

$$F_{in}^\ell(r) = \begin{cases} \frac{1}{1-\beta\lambda}\left(1 - a_j \frac{\overline{r}-c_\ell}{r-c_\ell} - [\beta\lambda - \alpha_j]\frac{c_h - c_\ell}{r-c_\ell}\right) & \textit{if } \alpha_j \le \beta\lambda \\ \frac{1}{1-\beta\lambda}\left(1 - a_j \frac{\overline{r}-c_\ell}{r-c_\ell}\right) \quad \textit{and} \quad \overline{\rho_i}^\ell = \frac{\alpha_j - \beta\lambda}{1-\beta\lambda} & \textit{otherwise} \end{cases}$$

$$F_{in}^h(r) = \begin{cases} 1 - \frac{\alpha_j}{\beta\lambda}\frac{\overline{r}-c_h}{r-c_h} \quad \textit{and} \quad \overline{\rho}_i^h = \frac{\alpha_j}{\beta\lambda} & \textit{if } \alpha_j \le \beta\lambda \\ 0(\textit{degenerate}) \quad \textit{and} \quad \overline{\rho}_i^h = 1 & \textit{otherwise.} \end{cases}$$

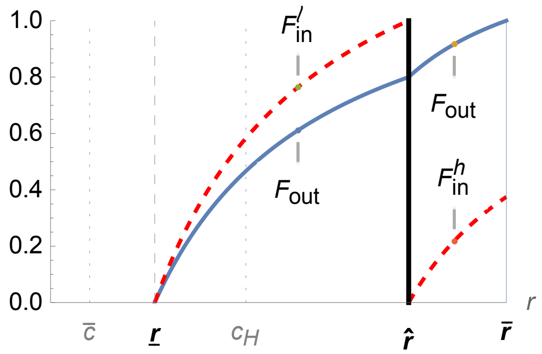
The outsider bank $(-j)$ mixes over $[\underline{r}, \overline{r}]$ according to distribution F_{out}:

$$F_{out}(r) = \begin{cases} \frac{1}{1-\alpha_j}\left(1 - \alpha_j \frac{\overline{r}-c_\ell}{r-c_\ell} - [\beta\lambda - \alpha_j]_+ \frac{c_h-c_\ell}{r-c_\ell}\right) & \textit{if } r \le \hat{r} \\ \min\{\frac{1}{1-\alpha_j} - \frac{\alpha_j}{1-\alpha_j}\frac{\overline{r}-c_h}{r-c_h}; 1\} & \textit{if } r > \hat{r} \end{cases}.$$

where $[x]_+$ *denotes* x *if* $x > 0$, *0 otherwise.*

Proof. See Appendix B.1 which also analytically derives the values of $\underline{r}$ and $\hat{r}$. ∎

Figure 3: Credit fee dispersion – pricing distributions
Naivety and adverse selection



Pricing cumulative distributions for the insider bank (red, dashed) for high- and low risk types, and for the outsider bank (blue, solid).

We will use Proposition 3 to establish the behaviour of the expected price for credit as a function of the signal triggered (ℓ or h) in the discussion ahead.[21] The equilibrium price dispersion is depicted in Figure 3 for the special case of a fully informative risk signal ($\lambda = 1$). The figure demonstrates how the insider bank discriminates between customers based on their riskiness, high credit-risk borrowers paying more. Given the price discrimination of the insider bank, the outsider bank appreciates that if she charges a fee at or above $\hat{r}$ then she will fail to attract all low-risk sophisticated customers from the insider bank.

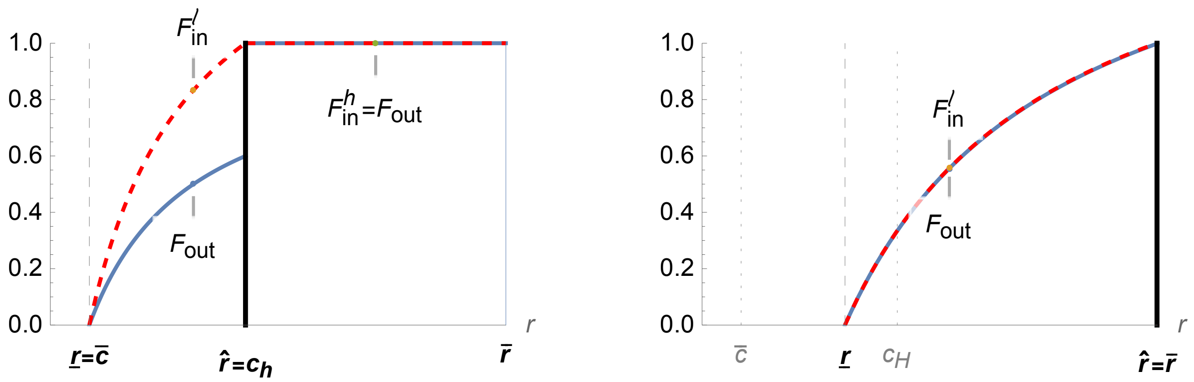
Figure 4 illustrates that the equilibrium approaches simpler models as limiting cases. Panel (a) depicts the limit as $\alpha \to 0$, the case with no naive consumers and adverse selection only. The lower-bound of the distributions approach the weighted average cost $\bar{c}$, while the upper bound of F^{ℓ}_{in} approaches $c_h(= c_H)$. At the same time, the distributions on the upper range (for $r \in [c_H, \bar{r}]$) converge to a mass-point on c_H for both the insider and the outsider banks. This demonstrates that an outsider bank winning a client with an h signal will make a loss as the credit fee which will have been charged will be below cost. This reflects the implication arising from the extant literature that, if adverse selection is the only friction then one cannot explain the profitability of banks who lend to high risk borrowers (pay day loans etc).

[21]See, in particular, the text around Proposition 6.

Panel (b) of Figure 4 illustrates the case in which $\alpha \to \beta\lambda$, so that high risk and naivety are equally prevalent. The figure shows that the upper parts of the distributions disappear and F_{in}^{h} becomes degenerate. The insider bank chooses to serve the naive high risk customers at a high price, and this pulls the outsider price offers upward. Notice that while in both limiting cases the pricing distributions are qualitatively similar (as the h-offer becomes degenerate), the presence of naivete shifts the domains upwards. This is the root cause of the profit-leakage phenomenon: when naivete is present equilibrium induces price dispersion at higher prices than what would happen otherwise. This raises profit for both the insider and the outsider bank. Such profit leakage has a crucial role in determining the allocation of industry profits in credit markets, and in turn, the prevailing prices of current accounts.

Figure 4: Credit fee dispersion - limiting cases
Naivety and adverse selection

(a) $\alpha \to 0$ (b) $\alpha \to \beta\lambda$



Panel (a) illustrates the benchmark case with no naive customers and adverse selection only. Panel (b) shows that when high risk and naivety are equally prevalent, the two distributions coincide and insider's high-type distribution becomes degenerate.

Formally, the bank's total second-period equilibrium profit can be decomposed into profits from its role as outsider and as insider bank. The insider profit is further decomposed into profits from high-risk signal type and from low-risk signal consumers. That is, the profit of a bank j with market share $\ell^{(j)}$ from the 1st period is:

$$\pi^{j} = \ell_j \pi_{in}^{\ell(j)} + \ell_j \pi_{in}^{h(j)} + \ell_{(-j)} \pi_{out}^{(j)}, \tag{12}$$

where analytical expressions of such components are identified in the proof of Proposition 3. Notice that the subscript of the market share for a bank's outsider role is $(-j)$, as it draws from the other bank's customers as an outsider bank.

5.2 First-period equilibrium

Naive customers' PCA decision is based on first period prices and so is given by the standard Hotelling threshold:

$$\hat{\gamma}^N = \frac{1}{2} + \frac{p_B - p_A}{2\tau}. \tag{13}$$

Sophisticated customers correctly predict the equilibrium market shares given their candidate threshold strategy, that is $\hat{\gamma}^S$. In turn, this pins down the parameter α_j which is required to calculate second period equilibrium credit fees. This allows us to calculate customers' expected payment conditional on every sophisticated customer following the equilibrium strategy. We will show that the reasoning discussed in the special case of no adverse selection extends to this setting.

Establishing the equilibrium consists of two steps. First, we show that for any, not necessarily equal[22] values of p_A and p_B, sophisticated and naive customers following the exact same threshold-strategy in the first stage is an equilibrium of the subgame. The argument is similar to the one employed in Section 3. We assume that low-risk types and high-risk types make identical first-period choices. This is justified, as customers do not know ex-ante whether they will be perceived as low risk signal or high-risk signal types by the bank, as that would require them to know the bank's screening technology and behavioural scoring system, which is unlikely. Therefore, for any strategy of sophisticated customers, the fraction of low and high types will be the same within the two banks. In addition, if sophisticated customers follow the same strategy as naive customers, the mass of naive customers will also be the same across both banks: $\alpha_j = \alpha_{-j} = \alpha$. As the two banks have the same structural parameters, and credit pricing is scale-free in the sense that it is independent of market share, sophisticated customers predict the same equilibrium expected credit fee from either bank. Therefore, sophisticates will base their decision solely on the observed first-period prices, and follow the same strategy as naive customers, justifying this as an equilibrium action.

Next, taking this behaviour as given, we look at the banks' total profit function across both periods, and calculate the first-order condition for the optimal PCA-price. We show this now for the case $\alpha < \beta\lambda$, and solve the complementary case in Appendix B.2. The

[22] such that $\hat{\gamma}^N \in (0, 1)$ is maintained.

overall profit-function encompassing both periods can be constructed using Equation (12). For Bank A for example, after substituting $\alpha_j = \alpha$, $\beta_j = \beta$ and $l_j = \hat{\gamma}$ to the bank's profit function :

$$\Pi_A = p_A\hat{\gamma} + \hat{\gamma}\eta(\lambda\beta(1-\beta)(c^H - c^L) + \alpha(\bar{r} - c_H) + (1-\hat{\gamma})\eta\left[\alpha(1-\alpha)(\bar{r} - c_H)\right]$$

We substitute $\hat{\gamma} = \gamma^N$ and calculate the best response by Bank A to any p_B as a solution to the following first-order condition:

$$\frac{\partial \Pi_A}{\partial p_A} = \frac{1}{2\tau}\Big(\tau - 2p_A + p_B - \beta\lambda(1-\beta)\eta(c_H - c_L) - \alpha^2\eta(\bar{r} - c_H)\Big) = 0$$

this gives the best-response function:

$$\tilde{p}_A(p_B) = \frac{1}{2}\Big(\tau + p_B - \beta\lambda(1-\beta)\eta(c_H - c_L) - \eta\alpha^2\left(\bar{r} - c_H\right)\Big)$$

The symmetric equilibrium is given by the fixed-point of this mapping, and we have:

Proposition 4 *There exists a unique symmetric equilibrium with the following first-period prices.*

$$p^\star = \begin{cases} \max\{\tau - \eta\left[\alpha^2(\bar{r} - c_H) + \beta\lambda(1-\beta)(c_H - c_L)\right]; 0\} & \textit{if } \alpha \leq \beta\lambda \\ \max\{\tau - \eta\Big[\alpha^2(\bar{r} - c_H) + (\alpha(\alpha - \beta\lambda) + \beta\lambda(1-\alpha))\frac{1-\beta}{1-\beta\lambda}(c_H - c_L)\Big]; 0\} & \textit{if } \alpha > \beta\lambda \end{cases} \tag{14}$$

Proof. *Appendix B.2.* ■

Proposition 4 offers the solution to a general model of banking and credit which combines naivety and adverse selection. The solution's simplicity can disguise the delicacy of the analysis required to establish the result. We discuss the implications of this proposition in the PCA and credit markets next.

6 Analysis: Free banking, AI and Cryptocurrencies.

We have solved a full model of competition which embeds customer naivety, adverse selection due to client risk, and the banking industry's screening technology. This detailed analysis allows us to explore three questions which are new to the literature:

1. Can naivety or adverse selection alone rationalise why some countries have widespread free banking, while others do not?

2. What are the consumer surplus implications of more naivety in the population?

3. Do consumers benefit if more transactions are hidden by cryptocurrencies, or if banks successfully deploy AI to make more accurate predictions as to their customers' type?

6.1 Why do we see free-banking in some countries and not in others?

Simple answers to this question are elusive. For example, the regulatory attention paid to protect retail banking is high in the US, UK and EU with no obvious differences in approach.[23] As we noted above, a first answer to this question might build on the seminal insights of Gabaix and Laibson (2006). Namely that naive consumers are valuable for a bank to acquire as they will not shop around when they subsequently need credit and so are profitable purchasers of credit products. Competition in the first market to acquire customers, via PCAs, drives the price of accounts down to zero: free banking.

Gabaix and Laibson (2006) did not allow for competition in the second stage. Proposition 2 extends the model to allow for such secondary market competition. Competition in the credit market creates a profit leakage to the outsider bank which in turn reduces the incentive to capture first period market share. But Proposition 2 shows that it does not eliminate it.

The evidence cited on the distribution of financial literacy across the world argues against naivety as being the sole determinant of banking prices. Von Thadden (2004) and authors following him focused instead on heterogeneity in risk. Von Thadden (2004) studied competition in the credit market, but without any client naivety, and the working assumed the insider received a perfectly informative signal as to client risk ($\lambda = 1$). In addition the work did not consider a first period of PCA competition.

Proposition 4 with $\alpha = 0$ completes the Von Thadden (2004) agenda. In equilibrium only the insider bank would profit from her information advantage, the outsider bank would obtain zero profit. It does indeed follow that a bank would compete more strongly

[23]To illustrate, the EU actively monitors retail banking competition (see press release MEMO/07/40 of the European Commission); active steps are taken to improve the market in the UK (e.g. the discussion of PS19/25 in footnote 8); and the US DoJ is actively exploring if more could be done to make US consumer banking more competitive (DoJ press release 21-1262).

in the first period to acquire market share as she is unable to profit from her rival in the credit competition when adverse selection is the only feature allowed for. This competition for first period market share effect pushes the PCA prices downward, making the free banking outcome more likely.

The problem with this explanation of free-banking is the concomitant prediction that the outsider bank makes a loss in expectation on every loan to a high risk borrower. When there is no naivety then all clients shop around. The insider bank offers credit to the high risk borrowers (that is the borrowers who have generated a high-risk signal h) at the break even level c_h. The outsider bank offers a weakly lower interest rate.[24] This implies that high risk clients an outsider bank wins must be loss-making in expectation. This prediction sits poorly with the prevalence of pay day loan companies and auto loans targeting those with precarious incomes.

This profit paradox, which is present in the Von Thadden (2004) inspired approach of adverse selection, is solved if one introduces some client naivety. The model had not been solved in the prior literature – we do so here. Adding naivety creates an incentive for insider banks to price credit even higher to their high risk clients. This in turn opens space for the rival bank to offer credit to high risk clients which remains profitable.

We capture the above discussion in the following proposition:

Proposition 5

1. *In the absence of adverse selection between clients, $\beta \in \{0,1\}$, PCA prices are weakly declining in the proportion of naive customers in the population, α.*

2. *In the absence of naivety, high risk clients won by an outside bank are loss making in expectation, and the outside bank makes zero economic profit in expectation.*

3. *There exists some level of naivety, $\hat{\alpha} > 0$, such that for all $\alpha > \hat{\alpha}$ the outside bank makes positive profit on high risk clients who switch from the insider.*

Proof. Part 1 follows from (14) in Theorem 4. For Part 2 and 3, see Appendix B.3. ■

Thus naivety ($\alpha > 0$) is required as well as adverse selection ($\beta \notin \{0,1\}$) to explain free banking across countries. If uncertainty as to the borrowers' credit riskiness is greater in country A than in country B, our analysis would imply that free-if-in-credit retail banking would be more likely in country A also. As we argued above, there is evidence that the UK and US have some of the lowest levels of job protection in the developed world, and have

[24]See panel (a) of Figure 4 and associated discussion.

households who spend a high proportion of their income (US) or seemingly even more than their income (UK). It follows that free banking could be explained in the UK, and its absence explained in France/Germany. While some customer naivety would explain why lending to high risk borrowers remains a profitable activity in all jurisdictions.

6.2 The Welfare effects of Naivety

6.2.1 Naivety and the price of credit

Suppose that a regulator or government agency could reduce the proportion of naive banking customers in the population. As noted above, the UK has been trying to do exactly this through new rules on clear client communication on borrowing linked to retail banking.[25] For the clients who were naive and became sophisticated then it would be natural to expect that they benefit. However such a change alters the whole equilibrium prices of accounts and of credit, making the outcome seemingly unclear.

Faced with fewer naive consumers in the population, each inside bank is less keen to charge high prices for credit as doing so risks losing too many sophisticated clients. This implies that the price distribution of the inside bank moves to lower prices in a first order stochastically dominant (FOSD) manner. While this suggests that credit conditions improve for the remaining naive types, the impact on sophisticated customers is less obvious. As the inside bank prices probabilistically lower to high types she increases the dispersion in her prices. The outsider optimally takes advantage of this to try and increase her margins if she is competing for a high risk consumer. At the same time, the outside bank wishes to price lower if she is competing for low risk customers. A more nuanced analysis is therefore required.

Proposition 6 *As naivety among customers decreases:*

1. *When $\alpha < \beta\lambda$ (few naives), then the expected price of credit decreases for both sophisticated and naive customers irrespective of the signal (h or ℓ) they generate.*

2. *When $\alpha \geq \beta\lambda$ (high levels of naivete), credit prices decrease for all types except for high-signal naive types. Expected credit prices to high risk signal types are constant in naivety, α.*

Proof. See Appendix B.4 ■

[25]See footnote 8.

If there are many naive clients ($\alpha \geq \beta\lambda$) then the intuition for the result above is easily explained. If the number of naive types declines, then the insider prices probabilistically lower for the $\ell-$signal types. There is no change in the price the insider charges to the high risk $h-$signal types as these are all charged at the upper bound $\bar{r}$. The outsider optimally responds to the changed $\ell-$signal pricing distribution by lowering her prices. All sophisticated clients therefore gain by receiving lower price offers for credit, and they can chose the best price from either bank. Naive $\ell-$signal types also benefit as the inside bank lowers her prices. But naive $h-$signal types remain stuck on the maximum price from the insider and so there is no change to their price for credit.

When there are few naive clients ($\alpha < \beta\lambda$) the proof is more nuanced, but the result holds. When there are fewer naive clients the inside bank lowers all her prices probabilistically, whether the signal was ℓ or h. The insider also expands (downwards) the range of credit prices she offers to high risk $h-$signal clients. The outside bank lowers some prices in responding to the insider, but she raises other prices to match the increased dispersion of high insider prices targetting $h-$signal customers. Overall however the outsider is responding to the insider and therefore a form of strategic complementarity in pricing remains. The outsider's price moves are not large enough to alter the direction of the effect set by the insider. So lower expected credit prices follow from a reduction in the number of naive consumers.

6.2.2 Naivety and consumer surplus

Fewer naive consumers may lower credit prices, but it doesn't follow that it improves market outcomes. There is a countervailing effect. As the prices in the market for credit decline, the value of incumbency diminishes. In turn this causes the banks to compete less aggressively in the first period for clients. It follows that the price of PCA banking rises. To establish the overall effect on consumer surplus we must combine these two effects.

To do so, we note that due to the full service assumption, welfare is constant. Hence any increase in industry profits is equivalent to an equal drop in consumer surplus. In contrast to the literature, we show that second period profits are only partially competed away in first-period markets, even if the lower boundary $p = 0$ does not bind. This is related to the profit leakage phenomenon described above. We have:

Proposition 7 *Consumer surplus has a global maximum at $\alpha = 0$ (assuming $\beta < 1$). Consumer surplus is continuous and piecewise convex in α over the ranges $\alpha \leq \beta\lambda$ and*

$\alpha > \beta\lambda$. *Furthermore:*

1. *In the case of 'free banking' ($p = 0$, US/UK markets): Consumer Surplus is monotonically declining in naivety α.*
2. *In the case of paid banking (European markets), Consumer Surplus has a global minimum at some $\alpha \in (0, 1)$.*

Proof. Appendix B.5. ■

Part 1 of Proposition 7 confirms that, in jurisdictions in which free banking is the norm, such as the UK and US, then any reduction in naivety improves consumer surplus overall. Proposition 6 established that such a change in client characteristics would lower the expected cost of credit. In turn this makes consumers less valuable to the banks and so they do not need to compete so hard in the PCA market. However, in the case of free banking, PCA prices are at their lower bound. Prices have been driven there by a combination of high competition, enough naive consumers, and sufficient mix of high and low risk clients. Reductions in naivety which don't turn the dial on PCA pricing are therefore consumer surplus improving.[26]

When free banking is not the norm (e.g. in France, Germany, Italy) lowering naivety in the population can be harmful to consumer surplus, at least initially. If one could get to *all* clients being sophisticated then consumer surplus would be maximised. But at intermediate levels of naivety, greater sophistication can lower consumer surplus. This is due to reduced profit on the credit market translating into yet higher prices in the PCA market. This is shown graphically in Figure 5 (panel (a)).[27].

It might seem strange that first period prices are positive and yet firms don't compete to return all rents to the consumers, so leaving consumer surplus unaffected. The reason this doesn't happen is because outsider banks make a positive profit due to client naivety. It is only the incremental profit between the outsider and the insider which is returned to consumers in the form of lower first period prices. Outsider bank profits are not competed away in the first stage and so affect overall consumer surplus.

Outside bank profits increase in the proportion of naive consumers α, as this raises the insider's prices. But the outsider's profits also increase in the mass of sophisticated customers $(1 - \alpha)$ as this is the addressable market which might switch to the outsider.

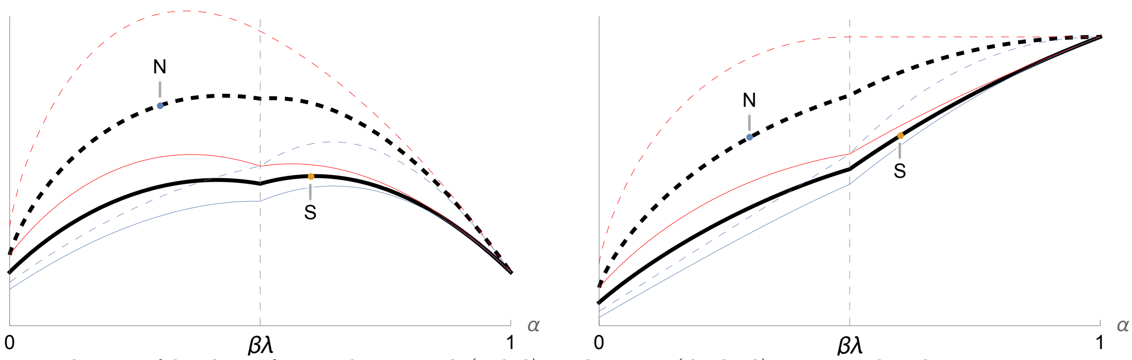
[26]This therefore provides further justification for the transparency drive the UK authorities have pursued with regards to over-draft pricing (see footnote 8 regarding FCA PS19/25).

[27]For the purpose of this illustration the expected value of the relevant fee distributions for all types was calculated. We report related algebra in the Internet Appendix.

The result is that the outsider profit is a negative quadratic in α, which explains why the consumer surplus is convex and why an interior optimum exists in the case of paid banking.

Figure 5: Total cost of banking

(a) *Paid banking* (b) *Free banking*



Total cost of banking for sophisticated (solid) and naive (dashed) types. Thin lines represent high-risk (red) and low-risk (blue) customers. Panel (a): $p = 0$ is non-binding (high τ), panel (b) low τ, where $p = 0$ is binding for all α.

6.3 Cryptocurrencies and AI

The quality of the inferences a bank can make as to its clients' credit risk is clearly important in understanding the price of credit and the overall cost of banking. As we noted above, the increasing use of Big Data and of AI algorithms improves the information advantage a bank can capture from its banking relationships. However, clients also have an increasing number of options for paying for goods outside of the banking system – cryptocurrencies being a good example. Increased use of these channels would deny the bank information and so weaken the quality of the signal.

We begin by establishing the main result. When AI is effective and widespread the credit risk technology is good, i.e. λ is high. Further improvements in AI are captured by λ increasing yet higher. Whereas if crypto use is widespread, stymieing the banks' credit risk technology, then λ is low. Further use of crypto would be captured by λ falling. We will see that the two regimes are separated at the threshold $\lambda = \frac{\alpha}{\beta}$ which is the ratio of the proportion of naive to the proportion of high credit risk types in the population.

Proposition 8

1. *In 'free banking' markets (UK, US):*

   (a) *When AI is dominant ($\lambda > \frac{\alpha}{\beta}$), customer surplus is decreasing in λ (better AI).*

   (b) *When cryptocurrency usage is dominant ($\lambda < \frac{\alpha}{\beta}$), customer surplus is increasing in λ (less crypto).*

2. *In 'paid banking' markets (EU):*

   (a) *When AI is dominant ($\lambda > \frac{\alpha}{\beta}$), customer surplus is unaffected by the level of λ (better or worse AI).*

   (b) *When cryptocurrency usage is dominant ($\lambda < \frac{\alpha}{\beta}$), customer surplus remains increasing in λ (less crypto).*

Proof. Appendix B.6. ■

When cryptocurrencies are widespread then further increasing the amount of crypto use (lower λ) decreases consumer surplus in both free banking and paid banking markets. Cryptocurrency use is always beneficial for a client who would generate a high risk $h-$signal. By denying their bank that signal they secure credit at lower prices. Proposition 8 shows this creates a negative externality on other clients which reverses the Consumer Surplus effect. To compensate for the worsening pool of ℓ-signal types, inside banks raise the cost of credit to $\ell-$signal clients. Therefore credit becomes more expensive. In the case of free banking, there are no further repercussions for PCA prices and so consumer surplus gets pulled down.

In the case of paid banking markets the result follows too as the higher cost of credit is not due to a reduction in the competitive pressure in the second stage. Rather it is due to a reduction in the quality of the screening technology which pushes up the expected cost of serving an $\ell-$signal customer. The higher credit prices are not therefore linked to higher profits, so there is nothing to be competed back in the first stage. Hence consumer surplus declines in the case of paid for banking too.

We now turn to the case in which AI is widespread ($\lambda > \alpha/\beta$). Proposition 8 determines that better AI would lower consumer surplus in the US/UK style free banking markets. This is because better technology to set apart risky and non-risky borrowers by the insider increases the adverse selection problem faced by the outsider bank. This forces the outsider bank to beware the winner's curse, and so the outsider prices credit less generously to consumers. In turn this relaxes the competitive pressure on the insider.

More AI therefore increases the price of credit. This is a profit gain for the inside bank as the outsider is hamstrung by the winner's curse. But as we noted above, in free banking markets there is no room for banks to compete these profits back to consumers through lower PCA prices in the first stage; PCA prices are already at their lower bound of zero. Hence consumer surplus declines. In paid banking markets the second period profits are competed back to consumers in the first period PCA prices, and so consumer surplus is unaffected by greater AI use in the AI dominant case.

Proposition 8 identifies a stark contrast as the derivative $\partial \mathrm{CS}/\partial \lambda$ changes sign between the crypto dominant and AI dominant settings. In the Introduction we noted this regime change which caused more AI and more cryptocurrency to both damage consumer surplus, even though the former improves information and the second harms it. We explore this regime change now.

Consider first the case when λ is below the cutoff $\left(\lambda < \frac{\alpha}{\beta}\right)$. This is the case when crypto is dominant and so, by a rearrangement, naive clients are relatively plentiful in the population. In this case the insider bank sets the maximum price to the high risk $h-$signal types she detects. Changes in crypto use, and so in the detection technology, do not alter the price distribution received by these high risk types. As a result changes in crypto use alter the expected cost of serving $\ell-$signal types, but do not impact the adverse selection problem for the outside bank – all sophisticated high risk types will seek to escape the guaranteed maximum price from the insider. So less crypto means lower expected costs for $\ell-$signal types for the outsider bank. This naturally pulls prices down and consumer surplus goes up.

By contrast, consider now the case when λ is above the cutoff $\left(\lambda > \frac{\alpha}{\beta}\right)$. This is the case when AI is dominant and so, again by a rearrangement, there are relatively few naive clients in the population. In this case, as we noted above, the insider expands (downwards) the range of credit prices she offers to the high risk $h-$signal clients as the inside bank cannot afford to allow all the sophisticated $h-$signal clients to leave. The proportion of such high risk clients the outsider gets is therefore no longer certain. Changes in technology λ therefore alter the strength of the adverse selection problem for the outsider. More AI (higher λ) makes the insider better at spotting the high-risk clients, and so increases the adverse selection problem for the outsider bank. This weakens the competitive constraint the outsider can offer, and so acts to pull consumer surplus down.

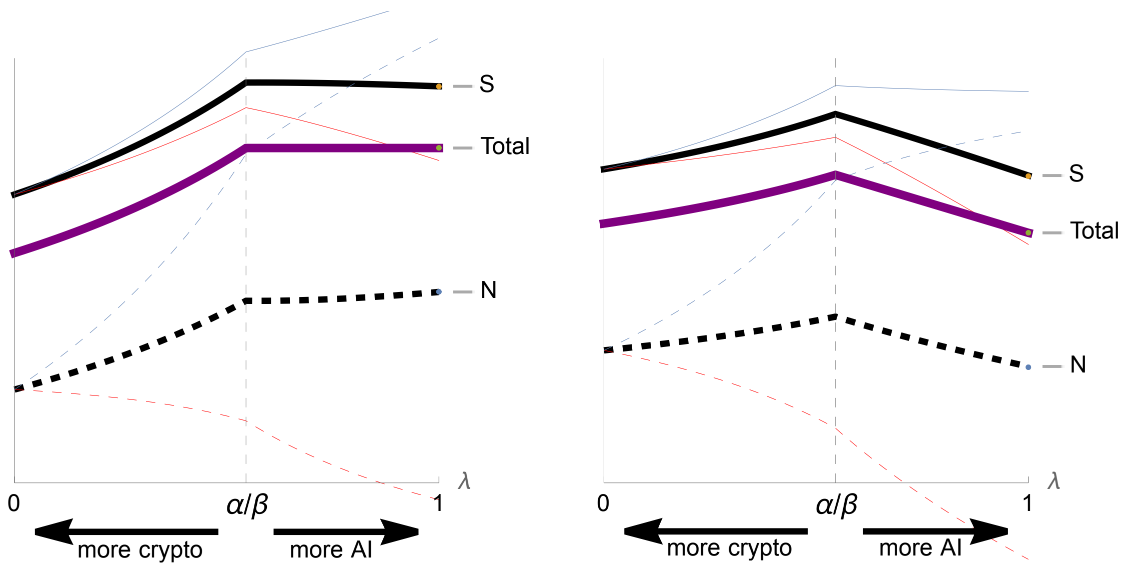
These aggregate consumer surplus movements hide substantial heterogeneity in banking fees, which we illustrate in Figure 6.[28] High risk naive consumers are the ones who

[28]The figure is based on calculations in the Internet Appendix

have most to lose from AI and most to gain from crypto use as they can hide their riskiness. For low risk naive consumers the insight is exactly reversed. The effect on sophisticated clients depends upon the banking regime.

Figure 6: Consumer surplus, impact of cryptocurrencies/AI

(a) *Paid banking* (b) *Free banking*



Panel (a) illustrates total consumer surplus for a case where $p = 0$ is non-binding, for high-risk (red) and low-risk (blue), naive (dashed) versus sophisticated (solid) customers. Panel (b) depicts the same for free banking, $p = 0$.

Our model sheds light on a second related question: will the increased use of cryptocurrencies, or the increased use of Artificial Intelligence, result in more or less free banking? Proposition 8 established the consumer surplus relationship between free banking and AI or crypto. This result does not however allow us to deduce the movement in the price of checking accounts as the banking industry we model contains credit as well as retail banking.

The equilibrium price of checking accounts was established in Proposition 4. The comparative static of retail banking prices therefore follows as a Corollary:

Corollary 1 (of Proposition 4)

1. *If AI is dominant ($\lambda \geq \alpha/\beta$) then increased use of AI leads to more free banking:*

$$\frac{\partial p^{\star}}{\partial \lambda} < 0. \tag{15}$$

2. *If cryptocurrency use is dominant ($\lambda < \alpha/\beta$) then increased use of crypto leads to less free banking (equation (15) holds).*

Proof. We use (14) and differentiate $p^\star$ with respect to λ. Establishing that (15) holds for the case $\alpha \leq \beta\lambda$ is immediate. Establishing (15) for $\alpha > \beta\lambda$ requires some standard working to establish that

$$\frac{\partial p^\star}{\partial \lambda} =_{\text{sign}} -(1-\alpha)^2\beta < 0.$$

The result then follows. ∎

When AI is dominant we noted in the intuition to Proposition 8 that further improvements in AI caused profits at the insider bank to rise as the winner's curse dampened the competitive constraint from the outsider. It therefore follows that having checking account customers is more valuable, and so the banks compete more aggressively to win banking clients in the first period. Hence improvements in AI pulls first period bank account prices down; we predict one would see more free banking.

When cryptocurrency use is dominant then the discussion after Proposition 8 explained why increased crypto use raised the costs of the banks in serving $\ell-$signal types and so lowered their profits. It follows that the profit benefit to an insider from receiving the signal on its customers declines. And hence the banks compete less aggressively to win the banking clients in the first period. This explains why greater crypto use results in higher first period prices, and so we predict one would see less free banking.

7 Conclusions

In this paper we have studied a model of competition between banks in the Personal Current Account market, and in the related market for consumer credit. In the credit market banks use overdrafts, credit cards and auto loans to serve their own customers and to win the business of customers who might not have a current account with them. We allow for differentiated competition, naive consumers, adverse selection and imperfect screening technology in the credit market.

Our analysis allows us to offer a new explanation as to why free banking is prevalent in some countries, while other countries have a positive charge for accounts. We confirm that consumer naivety can lead to such low initial prices then rip-off overdrafts, even though rival banks can seek to win customers in the credit market. We show however that adverse selection in borrower credit risk also leads to a similar effect. We show that alone neither mechanism can explain the apparent profitability of credit business as an outside provider and yet coincide with the available evidence on cross-country financial literacy. We provide the first theoretical analysis of a banking model with both naivety and information asymmetry.

We derive new implications with regard to customer welfare. Our model shows that improving naivety always improves consumer surplus in free banking markets, but not necessarily in paid banking markets. Finally, the analysis allows us to understand the implications of more crypto, or better artificial intelligence use in banking. We identify two distinct regimes. When customer naivety is the dominant friction, for example because intense cryptocurrency use prevents banks from identifying high-risk customers, less crypto use improves consumer surplus. When however information asymmetry between the banks dominates pricing choices, for example because of greater use of AI, banks can increase their profits on the market for credit. This however doesn't affect consumer welfare in paid banking markets: as all extra profit from greater AI use goes to insider, such incremental profits from incumbency are competed away on the markets for PCA.

Our modelling framework is also suited to the analysis of the potential impact of regulatory interventions, such as price-comparison websites, 'open banking', and switching services. Our model can also be used to explore strategic competition in AI technology between banks and the potentially resulting asymmetric equilibria. Such analyses, and further comparative exercise on cross-country differences are left to future work.

Appendix A Proofs, customer naivety

Appendix A.1 Proof of Proposition 1

Charging the maximum fee $\overline{r}$ is always a feasible strategy for both the insider and the outsider banks. Whenever the outsider charges $\overline{r}$, it loses the competition for sophisticated customers with certainty, and makes zero profit on lending. Therefore, in any equilibrium, the outsider bank is only willing to offer a fee which leads to a non-negative profit from the lending business, that is, $r_o \geq c$.

The insider bank is bounded by a similar incentive compatibility constraint. Charging $\overline{r}$ is always a feasible strategy, and even if at this fee it loses the competition for sophisticated customers with certainty, it obtains the following profit from its naive customers:

$$\underline{\pi} := \alpha \eta l (\overline{r} - c)$$

where we drop the subscript j (i.e. α_j) where possible without confusion. This quantity can be regarded as insider's minimax payoff: in any proposed equilibrium, the insider bank's profit from overdrafts must be at least $\underline{\pi}$. Now suppose that the equilibrium is such that for a sufficiently low offer r the insider wins the price competition with probability 1. Even in this case, the offer r must satisfy the inequality

$$\eta l \,(r - c) \geq \underline{\pi} \tag{A.1}$$

Let $\underline{r}$ denote the value of r which solves the equation corresponding to (A.1). We obtain:

$$\underline{r} := \alpha \overline{r} + (1 - \alpha) c \tag{A.2}$$

Equation (A.2) shows that $\underline{r}$ is a weighted average of the maximum fee and the break-even fee where the weights are the mass of naive (resp. sophisticated) customers. We proceed with a formal proof that there is no pure-strategy Nash-Equilibrium.

Lemma 4 *If $\alpha > 0$, no Pure-strategy Nash-equilibrium exists.*

Proof. Due to the incentive constraints the insider's offer must be such that $r \in [\underline{r}, \overline{r}]$, while the outsider's offer is $r_o \in [c, \overline{r}]$. Given $\alpha > 0$, we have $\underline{r} > c$. Then the outsider's best response to any offer r is $(r - \epsilon)$ for some $\epsilon > 0$. Insider's best response to any offer $r_o \in [\underline{r}, \overline{r}]$ is r_o, and to any offer $r_o < \underline{r}$ is $\overline{r}$. The mapping has no fixed point, so there is no pure strategy Nash equilibrium. ■

Next, we establish the unique equilibrium in mixed strategies through a series of Lemmas. Suppose that both outsider and insider mix according to CDFs F_{out} and F_{in}, with support $[\underline{F}_{out}, \overline{F}_{out}]$ and $[\underline{F}_{in}, \overline{F}_{in}]$ respectively. The following lemma establishes boundaries for the distributions:

Lemma 5 *The supports of the CDFs F_{out} and F_{in} must satisfy*

1. $\underline{F}_{out} = \underline{F}_{in} = \underline{r}$
2. $\overline{F}_{in} = \overline{r}$

Proof. We prove the Lemma through a series of claims.

Claim 1 *Insider will never offer any $r < \underline{r}$, so $\underline{F}_{in} \geq \underline{r}$. Furthermore, $Pr[r = \underline{r}] = 0$.*

(i) Charging $\overline{r}$ is always a feasible action for the insider, and even if it wins at some $r < \underline{r}$ with probability 1 and loses at $\overline{r}$ with probability 1, the latter still gives higher profit by the definition of $\underline{r}$. (ii) Suppose that there is a mass point at $\underline{r}$ by the insider, that is, $Pr[r = \underline{r}] > 0$. This can only be an equilibrium if $\underline{F}_{out} \leq \underline{r}$, otherwise insider would have incentives to increase the price. In addition, insider must win at $\underline{r}$ with certainty, otherwise it would find it better to charge $\overline{r}$ by the definition of $\underline{r}$. This implies outsider loses at $r_o = \underline{r}$ with a strictly positive probability. In that case, outsider is better off by charging $\underline{r} - \epsilon$ with probability 1, winning with certainty, and increasing its profit. Insider would therefore lose at $\underline{r}$. Contradiction to equilibrium.

Claim 2 *Outsider will never offer any $r < \underline{r}$, so $\underline{F}_{out} \geq \underline{r}$.*

Suppose the fee is $r < \underline{r}$. Because of claim 1, outsider wins with certainty, but then it would be better off by asking $\frac{\underline{r}+r}{2}$. Contradiction to equilibrium.

Claim 3 *Whenever $\underline{r} > c$ outsider makes strictly positive profit in equilibrium.*

For any offer $r_o \in (c, \underline{r})$ outsider wins with certainty and makes positive profit. As this is a feasible deviation, there must be positive profit in equilibrium.

Claim 4 *Outsider never places positive mass on any $r_o \geq \overline{F}_{in}$. In particular, $Pr[r_o = \overline{F}_{in}] = 0$.*

In this region the outsider would lose with certainty, implying zero profit and contradicting Claim 3.

Claim 5 *$\overline{F}_{in} = \overline{r}$ and insider's profit is $\alpha l \eta (\overline{r} - c)$*

Claim 4 implies that insider loses the bid with probability 1 at $\overline{F}_{in}$. Therefore, the insider's profit when playing $\overline{F}_{in}$ can be at most $\alpha l\eta\left(\overline{F}_{in} - c\right)$. Because the minimax payoff is $\alpha l\eta\,(\overline{r} - c)$, and the profit is increasing in r, it follows immediately that $\overline{F}_{in} = \overline{r}$ and insider's profit throughout the mixture is $\underline{\pi} = \alpha l\eta\,(\overline{r} - c)$.

Claim 6 *Outsider's lower boundary must be exactly* $F_{out} = \underline{r}$

Suppose that $\underline{F}_{out} > \underline{r}$. Then there exist a strategy for the insider to bid $\underline{F}_{out}$, win the competition with probability 1, and obtain a profit of $(\underline{F}_{out} - c)\,l\eta > \underline{\pi}$. This contradicts Claim 5. The result then follows using Claim 2.

Claim 7 *Insider's lower boundary must be exactly* $\underline{F}_{in} = \underline{r}$

Suppose $\underline{F}_{in} > \underline{r}$. Then the outsider could win all customers at $r_o = \underline{F}_{in} - \epsilon$, and would never ask anything below. Insider would then find it profitable to undercut this by asking $\underline{F}_{in} - \epsilon$. The claim follows using Claim 1.

Claim 8 *The constant profit to outsider over the mixture is* $\pi_{out} = (1-\alpha)l\eta\,(\underline{r} - c)$.

At $\underline{r}$ outsider wins with probability 1, because there is no mass by the insider.

This concludes the proof of the Lemma. ■

Next, we establish the equilibrium in mixed strategies. The expected payoff from any action which is played in a mixed strategy equilibrium must be equal (indifference condition). First, we apply this indifference condition to outsider's strategies to derive F_{in} (Lemma 6), then apply it to the insider's strategies to derive F_{out} (Lemma 7).

Lemma 6 *The insider bank mixes according to continuous distribution* $F_{in}(r)$ *with support* $[\underline{r}, \overline{r}]$, *and places a probability mass* α *on* $\overline{r}$, *where*

$$F_{in}(r) = 1 - \alpha\frac{\overline{r} - c}{r - c}. \tag{A.3}$$

Proof. There is no probability mass by the insider on $\underline{r}$ (Claim 1). Hence if the outsider sets $r_o = \underline{r}$ then all sophisticated customers switch with probability 1, and the outsider bank's profit is

$$\underline{\pi}_{out}(\underline{r}) = (1-\alpha)\,(\underline{r} - c)\,l\eta \tag{A.4}$$

For any higher bid $r_o > \underline{r}$ it must be that

$$\underbrace{Prob(r_o < r)}_{\text{O wins}} * \underbrace{(1-\alpha)l\eta\,(r_o - c)}_{\text{profit|win}} + \underbrace{Prob(r_o \geq r)}_{\text{I wins}} *0 = \underline{\pi}_{out}(\underline{r}) \tag{A.5}$$

After substitutions:

$$(1 - F_{in}(r))\,(1-\alpha)\,(r-c)\,l\eta = (1-\alpha)\,(\underline{r}-c)\,l\eta$$

After substituting the value of $\underline{r}$ we obtain A.3. This CDF satisfies $F_{in}(\underline{r}) = 0$ and $F_{in}(\overline{r}) = 1-\alpha$, which implies that insider is mixing over $[\underline{r}, \overline{r}]$ and places a probability mass of α on $\overline{r}$. ■

Lemma 7 *The outsider bank mixes according to continuous distribution $F_{out}(r)$ with support $[\underline{r}, \overline{r}]$ where*

$$F_{out}(r) = \frac{1}{1-\alpha} - \frac{\alpha}{1-\alpha}\frac{\overline{r}-c}{r-c}. \tag{A.6}$$

Proof. We know from Claim 4 in the proof of Lemma 5 that there is no probability mass on $\overline{r}$ by the outsider, implying

$$\underline{\pi}(\overline{r}) = \alpha\,(\overline{r}-c)\,l\eta \tag{A.7}$$

The indifference property implies

$$\underbrace{Prob(r \leq r_o)}_{\text{I wins}} \cdot \underbrace{(r-c)\,l\eta}_{\text{profit from all}} + \underbrace{Prob(r > r_o)}_{\text{O wins}} \cdot \underbrace{\alpha(r-c)l\eta}_{\text{profit from myopes}} = \underline{\pi}(\overline{r})$$

This leads to the following equality:

$$(1 - F_{out}(r))\,(r-c)\,l\eta + F_{out}(r)\alpha\,(r-c)\,l\eta = \alpha\,(\overline{r}-c)\,l\eta$$

Simplifying yields to (A.6). Note that the CDF satisfies $F_{out}(\underline{r}) = 0$ and $F_{out}(\overline{r}) = 1$ ■
Theorem 1 follows immediately from Lemmas 6 and (7).

Appendix A.2 Proof of Lemma 1

Switching probabilities. Consider first the continuous part of the distributions without the mass point. From Proposition 1:

$$f_{in}(r) = F'_{in}(r) = \alpha \cdot \frac{\overline{r} - c}{(r-c)^2} \qquad \text{and} \qquad f_{out}(r) = F'_{out}(r) = \frac{\alpha}{1-\alpha} \cdot \frac{\overline{r} - c}{(r-c)^2}$$

where again we drop the subscript j where this can be done without confusion. The joint PDF due to the independence assumption is:

$$f_{io}(r, r_o) := f_{in}(r) \cdot f_{out}(r_o) = \frac{\alpha^2}{1-\alpha} \cdot \frac{(\overline{r} - c)^2}{(r_o - c)^2 (r-c)^2}$$

With the joint density f_{io} it is possible to write formally

$$\Pr^{\text{mix}}[r_o < r] = \int_{\underline{r}}^{\overline{r}} \int_{\underline{r}}^{r} f_{io} dr_o dr$$

The internal integral, with respect to r_o and using (A.2) is:

$$\int f_{io} dr_o = -\frac{\alpha^2}{1-\alpha} \cdot \frac{(\overline{r} - c)^2}{(r_o - c)(r-c)^2}$$
$$\therefore \int_{\underline{r}}^{r} f_{io} dr_o = \frac{\alpha}{1-\alpha} \frac{\overline{r} - c}{(r-c)^2} - \frac{\alpha^2}{1-\alpha} \frac{(\overline{r} - c)^2}{(r-c)^3}$$

After integrating each component we get:

$$\Pr^{\text{mix}}[r_o < r] = \frac{\alpha}{1-\alpha} (\overline{r} - c) \left[\frac{-1}{r-c} \right]_{\underline{r}}^{\overline{r}} - \frac{\alpha^2}{1-\alpha} (\overline{r} - c)^2 \left[\frac{-1}{2(r-c)^2} \right]_{\underline{r}}^{\overline{r}}$$

After substitution of the integral boundaries, we obtain the formula for the probability:

$$\Pr^{\text{mix}}[r_o < r] = \frac{1-\alpha}{2}$$

This probability only considers the mass over the continuous-part of the two distributions, so it gives the probability mass of winning when the insider mixes. In addition, the outsider wins with certainty whenever the insider plays $\overline{r}$. Together we obtain

$$Prob[r_o < r] = \frac{1-\alpha}{2} + \alpha = \frac{1+\alpha}{2} \qquad \text{and} \qquad Prob[r < r_o] = 1 - \frac{1+\alpha}{2} = \frac{1-\alpha}{2}$$

Expected fees. Notice first that sophisticated customers pay the *minimum of* r and r_o. Let $r_{min} := \min\{r, r_o\}$. With independent random variables it is straightforward to show that

$$F_{min}(r) = F_{in}(r) + F_{out}(r) - F_{in}(r)F_{out}(r) \tag{A.8}$$

Substituting the CDF's from Proposition 1 and integrating with respect to r one obtains:

$$f_{min}(r) = \frac{\alpha^2(2\overline{r} - c - r)(\overline{r} - c)}{(1-\alpha)(r-c)^3}$$

Expected fee paid by sophisticated customers follows as:

$$\mathbb{E}[r \mid \mathrm{S}] := \int_{\underline{r}}^{\overline{r}} r \cdot f_{min}(r) = c + \left[2\alpha + \frac{\alpha^2 \ln[\alpha]}{1-\alpha}\right](\overline{r} - c)$$

Naive customers always pay the insider's offer:

$$\mathbb{E}[r \mid \mathrm{N}] = \int_{\underline{r}}^{\overline{r}} r f_{in} dr + \alpha\overline{r} = c + \alpha(1 - \ln[\alpha])\,(\overline{r} - c)$$

Appendix A.3 Proof of Lemma 2

From the proof of Proposition 1 bank j's profit as insider and outsider is as follows:

$$\begin{aligned}
\pi_{in}^{(j)} &= \alpha_j l_j \eta(\overline{r} - c)\\
\pi_{out}^{(j)} &= (1 - \alpha_{-j})(\underline{r}_{-j} - c) l_{-j}\eta.
\end{aligned}$$

The outsider profit can be rewritten as

$$\pi_{out}^{(j)} = l_{-j}\eta(1-\alpha_{-j})\left(\alpha_{-j}\overline{r} + (1-\alpha_{-j})c - c\right) = l_{-j}\eta\alpha_{-j}(1-\alpha_{-j})\left(\overline{r} - c\right).$$

The overall profit is:

$$\pi^{(j)} = \pi_{in}^{(j)} + \pi_{out}^{(j)} = \eta\left(l_j\alpha_j + l_{-j}\alpha_{-j}(1-\alpha_{-j})\right)(\overline{r} - c).$$

Notice that as there are α naive consumers in the population,

$$l_j\alpha_j + (1-l_j)\alpha_{-j} = \alpha \quad \Rightarrow \quad l_j\alpha_j + l_{-j}\alpha_{-j} = \alpha \text{ and } \alpha_{-j} = \frac{\alpha - l_j\alpha_j}{1 - l_j},$$

therefore

$$\pi^{(j)} = \eta \left[\alpha - \frac{(\alpha - \alpha_j l_j)^2}{1 - l_j}\right] (\overline{r} - c) \text{ giving (4).}$$

In the text we consider two special cases. Firstly the setting in which the naive customers are evenly distributed among the two banks in the second stage, that is, $\alpha_A = \alpha_B = \alpha$. Substituting in (4) we see that the bank's profit in the second stage is:

$$\pi^{(j)} = \left[\alpha(1 - \alpha) + l_j \alpha^2\right] (\overline{r} - c) \tag{A.9}$$

Now consider the even more special setting in which the market shares are equal, that is $l_A = l_B = 1/2$. The expected profits of each bank are now:

$$\pi^{(A)} = \pi^{(B)} := \alpha \left(1 - \frac{\alpha}{2}\right) (\overline{r} - c)\, \eta \tag{A.10}$$

Appendix A.4 Proof of Lemma 3

When $\alpha_A = \alpha_B = \alpha$, using Lemma 1 the expected choice of second-period overdraft fees for sophisticated customers from their insider and outsider banks is independent of whether bank A or bank B is chosen. Therefore, if sophisticated consumers predict that in equilibrium $\alpha_A = \alpha_B$, they will base their decision only on first-period prices. Consequently, their decision will be identical to that of naive customers. It follows that sophisticated consumers will use a threshold strategy with $\hat{\gamma} = \hat{\gamma}^N$, which justifies the belief that $\alpha_A = \alpha_B$.

Appendix B Proofs, naivety and adverse selection

Appendix B.1 Proof of Proposition 3

Proposition 3 is proven in three parts. First, we describe posterior beliefs given the signal structure. Then, we characterize the structure of the equilibrium. Finally, we derive the fee distribution under this structure. In the proofs we drop the bank subscript j when this can be done without confusion. Related to this, all expressions which quantify one bank's profit from serving a certain type of customers (and so all indifference conditions) are scaled with the bank's market size l_i and the probability of liquidity shock η, which we suppress for easier readability of proofs.

Part 1: posterior beliefs.

Straightforward application of Bayes' theorem leads to the following posterior probabilities:

$$Pr[H|h] = 1 \qquad Pr[H|\ell] = \frac{\beta - \beta\lambda}{1 - \beta\lambda}$$
$$Pr[L|h] = 0 \qquad Pr[L|\ell] = \frac{1 - \beta}{1 - \beta\lambda}$$

The unconditional probabilities of signal arrivals are clearly

$$Pr[h] = \beta\lambda \qquad Pr[\ell] = 1 - \beta\lambda$$

These are also the (unconditional) probabilities of offering r_h and r_ℓ respectively by the insider bank. Conditional on the signal, the banks' assessment of the borrower's riskiness is as follows:

$$c_h := c_H$$
$$c_\ell := \frac{\beta - \beta\lambda}{1 - \beta\lambda} c_H + \frac{1 - \beta}{1 - \beta\lambda} c_L$$

So in the limit of fully informative signal ($\lambda \to 1$) we obtain $c_\ell = c_L$, in the limit of $\lambda \to 0$, we have $c_\ell = \bar{c}$.

Part 1: structure of equilibrium.

Claim 1 *The supports of insider's distributions for low and high signal cannot overlap, that is,* $\overline{F}_i^\ell \leq \underline{F}_i^h$.

Proof. Intuitively, the proof establishes that the insider cannot be indifferent between offering two distinct fees to high-type and to low-type customers at the same time, while facing the same outsider distribution. Suppose $\underline{F}^h < \overline{F}^\ell$. At any $r \in \left(\underline{F}^h, \overline{F}^\ell\right)$ the insider must be indifferent independently for the low-types and for the high-types. That means, for an arbitrary r and $q \in \left(\underline{F}^h, \overline{F}^\ell\right)$:

$$(1 - F_o(r))(1 - \alpha)(r - c_\ell) + \alpha(r - c_\ell) = (1 - F_o(q))(1 - \alpha)(q - c_\ell) + \alpha(q - c_\ell), \text{and}$$
$$(1 - F_o(r))(1 - \alpha)(r - c_h) + \alpha(r - c_h) = (1 - F_o(q))(1 - \alpha)(q - c_h) + \alpha(q - c_h).$$

After simplifications for each $\theta \in \{\ell, h\}$ we obtain:

$$(1 - F_o(r))(1-\alpha)(r - c_\theta) + \alpha(r - c_\theta) = (1 - F_o(q))(1-\alpha)(q - c_\theta) + \alpha(q - c_\theta)$$
$$\therefore [F_o(r)(r - c_\theta) - F_o(q)(q - c_\theta)] = \frac{r - q}{1 - \alpha}$$

From the last equation the contradiction is obvious, as the right-hand side is constant, while the left-hand side takes different values for high and low types. ■

Claim 2 *Insider's low and high distributions cannot be disjoint with a gap between the two intervals, i.e.* $\overline{F}_i^\ell \geq \underline{F}_i^h$.

Proof. Suppose that they are disjoint, $\overline{F}_i^\ell < \underline{F}_i^h$. There cannot be any probability mass by the outsider on any $r \in [\overline{F}_i^\ell, \underline{F}_i^h]$, as it would find it optimal to put this mass on $\underline{F}_i^h - \epsilon$ instead. However, this cannot be optimal for the insider. As the insider wins with the same probability over high types for every $[\overline{F}_i^\ell, \underline{F}_i^h)$, it would find optimal to move some probability mass to the left, and increase its payoff. If the outsider placed no probability mass on $[\overline{F}_i^\ell, \underline{F}_i^h]$ then the insider would find it optimal to move mass up to F_i^h. Contradiction to equilibrium. ■

Notice that Claim 1 and 2 together imply that $\overline{F}_i^\ell = \underline{F}_i^h$.

Claim 3 *The support of F_o coincides with the union of the supports of F_i^h and F_i^ℓ.*[29] *That is, $\underline{F_i^\ell} = \underline{F_o}$ and $\overline{F_i^h} = \overline{F_o}$*

Proof. The proof is analogous to previous results in Section 3. Whenever $\underline{F}_i^\ell < \underline{F}_o$, insider has incentives to put the probability mass on $[\underline{F}_i^\ell < \underline{F}_o)$ to $\underline{F}_o$ instead. Similarly, if $\underline{F}_i^\ell > \underline{F}_o$. The equality of upper boundaries can be seen analogously. ■

Claim 4 *There is no probability mass by the insider at the minimum boundary of both F_i^ℓ and F_i^h.*

Proof. Suppose there is strictly positive mass on $\underline{F}_i^\ell$ by the insider. Then the outsider loses at $\underline{F}_i^\ell$ with some positive probability. Instead of playing $\underline{F}_i^\ell$, outsider could put all probability mass to $\underline{F}_i^\ell - \epsilon$, and win with probability 1. Contradiction to equilibrium. Suppose there is strictly positive mass on $\hat{r}$ by the insider when targeting high type

[29]One must be careful with the mathematical language here, because the randomization does not necessarily happen over a compact interval. Notice however that the usual definition of the support of a distribution, *supp*(F) is *the closure* of the set of possible values with nonzero mesure. That is, $\overline{F} := \sup\{x : F(x) < 1\} \in supp(F)$ even if $\overline{F}$ is never played.

consumers. Then the outsider loses high types with some positive probability. Instead, it could place some mass at $\hat{r} - \epsilon$, win all high types, and not lose any on the low types. Contradiction to equilibrium. ■

Claim 5 *If the distributions are not degenerate (not a mass-point), only one of insider's minimax-profits can be binding.*

Proof.

To be specific, either $\underline{\pi}_i(\ell)$ or $\underline{\pi}_i(h)$ is binding, where

$$\underline{\pi}_i(h) = \alpha\beta\lambda\,(\overline{r} - c_h) \tag{B.11}$$

$$\underline{\pi}_i(\ell) = \alpha(1-\beta\lambda)\,(\overline{r} - c_\ell) \tag{B.12}$$

are profits the bank would obtain by serving naive customers only at the maximum price.

Consider the price $\hat{r} := \overline{F}_i^\ell = \underline{F}_i^h$ which is played in both ℓ and h distributions. The required probability mass on F_o to make the insider bank indifferent between playing $\hat{r}$ and their minimax payoff for low and high type respectively as follows:

$$\text{If B.11 binds: } (1 - F_o(\hat{r}))(1-\alpha)(\hat{r} - c_h) + \alpha(\hat{r} - c_h) = \alpha\,(\overline{r} - c_h)$$
$$\text{If B.12 binds: } (1 - F_o(\hat{r}))(1-\alpha)(\hat{r} - c_\ell) + \alpha(\hat{r} - c_\ell) = \alpha\,(\overline{r} - c_\ell)$$

As $c_\ell \neq c_h$, only one of these two can be binding. ■

Part 2: credit fee distributions. The insider randomizes independently for low signals and for high signals, while competing against a constant probability distribution F_o. For notational convenience let $F_o^\ell(r)$ and $F_o^h(r)$ refer to the lower ($r < \hat{r}$) and upper ($r > \hat{r}$) part of outsiders' distribution F_o. Suppose that $\underline{\pi}_i(h)$ is binding so that (B.11) holds. Then the bank is indifferent in offering any $r \in [\hat{r}, \overline{r}]$ if:

$$\left(1 - F_o^h(r)\right)(1-\alpha)\beta\lambda\,(r - c_h) + \alpha\beta\lambda\,(r - c_h) = \alpha\beta\lambda\,(\overline{r} - c_h)$$

This defines the upper part of the outsider's CDF:

$$F_o^h(r_h) = \frac{1}{1-\alpha} - \frac{\alpha}{1-\alpha}\frac{\overline{r} - c_h}{r_h - c_h} \tag{B.13}$$

Suppose that the cutoff-point between F_i^ℓ and F_i^h is some $\hat{r} \in (c_\ell, \overline{r})$. In what follows we write everything as a function of an arbitrary cutoff-value $\hat{r}$. Equilibrium is established by deriving outsider's profit at two different points in its strategy domain, $\underline{F}$ and $\overline{r}$ as a function of $\hat{r}$, and then solving for $\hat{r}$.

Consider F_i^ℓ first. The insider must be indifferent at any $r \in [\underline{F}, \hat{r}]$ for low-types:

$$\begin{aligned}\Big(1 - F_o^\ell(r)\Big)(1-\alpha)(1-\beta\lambda)\,(r - c_\ell) + \alpha(1-\beta\lambda)\,(r-c_\ell) \\ = \Big((1-\alpha)(1-\beta\lambda)(1-F_o^h(\hat{r}))(\hat{r}-c_\ell) + \alpha(1-\beta\lambda)(\hat{r}-c_\ell)\Big)\end{aligned} \tag{B.14}$$

Using (B.13), from B.14 we can express $F_o^\ell(r)$ as a function of the extra argument $\hat{r}$:

$$F_o^\ell(r, \hat{r}) = \frac{1}{1-\alpha} - \frac{\alpha}{1-\alpha}\frac{(\overline{r}-c_h)(\hat{r}-c_\ell)}{(\hat{r}-c_h)(r-c_\ell)} \tag{B.15}$$

Now we define the lower bound of the distribution as a function of $\hat{r}$ implicitly through:

$$F_o^\ell(\underline{r}, \hat{r}) = 0 \tag{B.16}$$

From (B.15) this gives the unique solution for the lower boundary $\underline{F}$:

$$\underline{r}(\hat{r}) = c_\ell + \frac{\alpha\,(\overline{r}-c_h)\,(\hat{r}-c_\ell)}{\hat{r}-c_h} \tag{B.17}$$

At the lower boundary the outsider wins with probability 1, so we can compute the outsider's profit at $\underline{F}$ as a function of $\hat{r}$:

$$\pi_o(\underline{r}(\hat{r})) = (1-\alpha)\left(c_\ell + \frac{\alpha\,(\overline{r}-c_h)\,(\hat{r}-c_\ell)}{\hat{r}-c_h} - (\beta\lambda c_h + (1-\beta\lambda)c_\ell)\right) \tag{B.18}$$

Whenever the outsider charges $\overline{r} - \epsilon$, with $\epsilon \to 0$, she obtains a profit of

$$\pi_o(\overline{r}) = \overline{\rho}_i(1-\alpha)\beta\lambda(\overline{r}-c_h) \tag{B.19}$$

where $\overline{\rho}_i$ is the mass point at $\overline{r}$ by the insider. The cutoff value $\hat{r}$ must also satisfy that at $\hat{r}$, F_i^h has no mass-point and the outsider obtains all h-types. So, the outsider is indifferent between playing $\overline{r}$ and $\hat{r}$ if

$$(1-\alpha)\beta\lambda(\hat{r}-c_h) = \overline{\rho_i}(1-\alpha)\beta\lambda\,(\overline{r}-c_h)\,.$$

Solving for the mass point we obtain

$$\overline{\rho}_i(\hat{r}) = \frac{\hat{r} - c_h}{\overline{r} - c_h} \tag{B.20}$$

Finally, the outsider must be indifferent between playing $\underline{r}$ and $\overline{r}$. Using (B.19) and (B.18) then $\hat{r}$ is determined by solving $\pi_o(\underline{r}, \hat{r}) = \pi_o(\overline{r}, \hat{r})$:

$$\hat{r}^{\star} = c_h + \frac{\alpha\,(\overline{r} - c_h)}{\beta\lambda} \tag{B.21}$$

Substituting (B.21) into (B.20) we see that the mass point is given by $\frac{\alpha}{\beta\lambda}$. Notice that as $\beta\lambda \to \alpha$ (from above) we have $\hat{r}^{\star} \to \overline{r}$, and so F_i^h becomes degenerate. We branch out the rest of the proof and investigate two cases, $\alpha < \beta\lambda$ and $\alpha \geq \beta\lambda$ separately.

Case 1: $\alpha < \beta\lambda$. Combining (B.21) with (B.17) gives the lower boundary:

$$\underline{r} = c_\ell + \alpha(\overline{r} - c_h) + \beta\lambda(c_h - c_\ell) \tag{B.22}$$

Substituting $\hat{r}^{\star}$ into the outsider profit function (B.19) gives

$$\pi_o^{\star} = \alpha(1 - \alpha)(\overline{r} - c_h). \tag{B.23}$$

Substituting $\hat{r}^{\star}$ into F_o^h in (B.13) gives the probability mass from the left of the threshold, that is, the probability that the insider loses the low-types if playing $\hat{r}$:

$$F_o^h(\hat{r}) = \frac{1 - \beta\lambda}{1 - \alpha}.$$

This implies that the insider wins with the complementary probability ($\frac{\beta\lambda - \alpha}{1 - \alpha}$) at $\hat{r}$. We can compute the insiders' profit on l-types as she plays $\hat{r}$:

$$\pi_i^\ell(\hat{r}) = (1 - \alpha)\frac{\beta\lambda - \alpha}{1 - \alpha}\,(\hat{r} - c_\ell) + \alpha(\hat{r} - c_\ell) = \alpha(\overline{r} - c_h) + \beta\lambda(c_h - c_\ell) \tag{B.24}$$

Notice that this equilibrium payoff exceeds the corresponding minimax payoff in Equation (B.12) whenever $\alpha \leq \beta\lambda$. This confirms that the minimax payoff for the high type is binding whenever $\alpha \leq \beta\lambda$. Using (B.24) we can write insider's indifference condition as

$$(1 - F_o^\ell(r))(1 - \alpha)(r - c_\ell) + \alpha(r - c_\ell) = \alpha(\overline{r} - c_h) + \beta\lambda(c_h - c_\ell)$$

This pins down the outsider's CDF over the interval $[\underline{r}, \hat{r})$.

$$F_o^\ell(r) = \frac{1}{1-\alpha} - \frac{\alpha(\overline{r} - c_\ell) + (\beta\lambda - \alpha)(c_h - c_\ell)}{(1-\alpha)(r - c_\ell)} \tag{B.25}$$

The insider's distributions are derived from the outsider's indifference condition where the outsider's equilibrium payoff is given by Equation (B.23). That leads to the conditions[30]:

$$\left(1 - F_i^\ell(r)\right)(1-\alpha)(1-\beta\lambda)(r - c_\ell) + (1-\alpha)\beta\lambda(r - c_h) = (1-\alpha)\alpha(\overline{r} - c_h)$$
$$\left(1 - F_i^h(r)\right)(1-\alpha)\beta\lambda(r - c_h) = (1-\alpha)\alpha(\overline{r} - c_h)$$

so the respective distributions are:

$$F_i^\ell(r) = \frac{1}{1-\beta\lambda} - \frac{\alpha(\overline{r} - c_h) + \beta\lambda\Delta c}{(1-\beta\lambda)(r - c_\ell)} \tag{B.26}$$

$$F_i^h(r) = 1 - \frac{\alpha}{\beta\lambda}\frac{\overline{r} - c_h}{r - c_h} \tag{B.27}$$

It is straightforward to verify that (i) $F_i^\ell(r) = 0$ and $F_o(r) = 0$ have the same solution; (ii) $F_i^\ell(\hat{r}) = 1$ and $F_i^h(\hat{r}) = 0$; (iii) $F_o(\overline{r}) = 1$, and (iv) $F_i^h(\overline{r}) = 1 - \frac{\alpha}{\beta\lambda}$, which corresponds to the mass point derived above.

Case 2: $\alpha \geq \beta\lambda$**.** From (B.21), the distribution F_i^h becomes degenerate and $r_h = \overline{r}$. The common lower bound of distributions F_o and F_i^ℓ are pinned down by insider's constraint that serving all customers at $\underline{r}$ must lead to as much profit as serving naives only at $\overline{r}$ (i.e. equation (B.12) is binding). From this:

$$\underline{r} = \alpha\overline{r} + (1-\alpha)c_\ell.$$

Because there is no mass-point by the insider at $\underline{r}$, if outsider plays $\underline{r}$ sophisticated customers switch with probability 1 and the bank obtains a profit of

$$\pi_o(\underline{r}) = (1-\alpha)\left(\beta\lambda(\underline{r} - c_h) + (1-\beta\lambda)(\underline{r} - c_\ell)\right). \tag{B.28}$$

[30] When outsider plays $\overline{r} - \epsilon$ $(\epsilon \to 0)$, it obtains all sophisticated $(1-\alpha)$, type-h customers (probability $\beta\lambda$) whenever the insider plays the mass-point (prob. $\alpha/\beta\lambda$), which gives the right-hand side.

For any $r > \underline{r}$ it must be the case that

$$(1-F_i^\ell(r))(1-\alpha)(1-\beta\lambda)(r-c_\ell)+(1-\alpha)\beta\lambda(r-c_h) = (1-\alpha)(\beta\lambda(\underline{r}-c_h)+(1-\beta\lambda)(\underline{r}-c_\ell)) \tag{B.29}$$

Solving for F_i^l we obtain:

$$F_i^\ell = \frac{1}{1-\beta\lambda} - \frac{\alpha}{1-\beta\lambda}\frac{\overline{r}-c_\ell}{r-c_\ell}. \tag{B.30}$$

This satisfies (by construction) $F_i^\ell(\underline{r}) = 0$. Solving $F_i^l(r) = 1$ for r gives:

$$r^{max} := \overline{r} + \frac{\alpha-\beta\lambda}{\beta\lambda}\left(\overline{r}-c_\ell\right),$$

from which it is clear that

$$r^{max} < \overline{r} \quad \Leftrightarrow \quad \alpha < \beta\lambda \quad \text{and} \quad r^{max} > \overline{r} \quad \Leftrightarrow \quad \alpha > \beta\lambda.$$

This implies that whenever $\alpha > \beta\lambda$, the upper boundary of the mixture is $\overline{r}$, and there is a mass-point by insider on $\overline{r}$. In this case the CDF at $\overline{r}$ is

$$F_i^\ell(\overline{r}) = \frac{1-\alpha}{1-\beta\lambda},$$

therefore, the probability mass on $\overline{r}$ must be

$$\rho_i = 1 - \frac{1-\alpha}{1-\beta\lambda} = \frac{\alpha-\beta\lambda}{1-\beta\lambda}. \tag{B.31}$$

Outsider will never bid above $\overline{r}$, so when insider's offer to both types is $\overline{r}$ then it serves only naive customers and obtains the following profit:

$$\pi_i(\overline{r}) := \alpha\left(\beta\lambda(\overline{r}-c_h)+(1-\beta\lambda)(\overline{r}-c_\ell)\right) = \alpha(\overline{r}-\overline{c}). \tag{B.32}$$

Her indifference condition is:

$$(1-F_o(r))((1-\alpha)(1-\beta\lambda)(r-c_\ell)) + \alpha(1-\beta\lambda)(r-c_\ell) = \alpha(1-\beta\lambda)(\overline{r}-c_\ell),$$

which leads to the outsiders' distribution:

$$F_o(r) = \frac{1}{1-\alpha} - \frac{\alpha}{1-\alpha} \cdot \frac{\overline{r} - c_\ell}{r - c_\ell}. \tag{B.33}$$

Solving $F_o(r) = 0$ trivially shows that $r = \underline{r}$, which implies that $F(\underline{r}) = 0$ and $F(\overline{r}) = 1$, so there is no mass-point in outsider's CDF as they mix over $[\underline{r}, \overline{r}]$.

Appendix B.2 Proof of Proposition 4

We restate the bank's profit functions for all cases based on the proof Appendix B.1.[31]

$$\begin{aligned}
\pi_{in}^h &= [\eta\ell_j] \cdot \alpha_j\beta\lambda(\overline{r} - c_h) \qquad \text{For both Case 1 (B.11) \& 2 ((B.32)).} \\
\pi_{in}^\ell &= \begin{cases} [\eta l_j] \cdot (1-\beta\lambda)\,[\alpha_j(\overline{r} - c_h) + \beta\lambda(c_h - c_\ell)] & \text{from (B.24)} \quad \text{if} \beta\lambda > \alpha \\ [\eta l_j] \cdot (1-\beta\lambda)\,[\alpha_j(\overline{r} - c_h) + \alpha(c_h - c_\ell)] & \text{from (B.32)} \quad \text{otherwise} \end{cases} \\
\pi_{out} &= \begin{cases} [\eta l_{(-j)}](1-\alpha_{(-j)})\left(\alpha_{(-j)}(\overline{r} - c_h)\right) & \text{from (B.23)if} \beta\lambda > \alpha \\ [\eta l_{(-j)}](1-\alpha_{(-j)})\left(\alpha_{(-j)}(\overline{r} - c_h) + [\alpha_{(-j)} - \beta\lambda]_+(c_h - c_\ell)\right) & \text{from(B.28)otherwise.} \end{cases}
\end{aligned} \tag{B.34}$$

The aggregate insider profit is (note that $c_h - c_\ell = \frac{1-\beta}{1-\beta\lambda}(c_H - c_L)$):

$$\begin{aligned}
\pi_{in} &= \begin{cases} [\eta l_j]\left(\alpha(\overline{r} - c_h) + \beta\lambda(1-\beta\lambda)(c_h - c_\ell)\right) & \text{if } \beta\lambda > \alpha \\ [\eta l_j]\left(\alpha(\overline{r} - c_h) + \alpha(1-\beta\lambda)(c_h - c_\ell)\right) & \text{otherwise.} \end{cases} \\
\therefore \pi_{in} &= \begin{cases} [\eta l_j]\left(\alpha(\overline{r} - c_H) + \beta\lambda(1-\beta)(c_H - c_L)\right) & \text{if } \beta\lambda > \alpha \\ [\eta l_j]\left(\alpha(\overline{r} - c_H) + \alpha(1-\beta)(c_H - c_L)\right) & \text{otherwise.} \end{cases}
\end{aligned} \tag{B.35}$$

For a symmetric equilibrium we substitute $\alpha_j = \alpha_{-j} = \alpha$ and $l_j = \hat{\gamma}$ and $l_{-j} = 1 - \hat{\gamma}$.

Case 1: $\alpha < \beta\lambda$: The profit functions from above:

$$\pi_A = p_A\gamma + \eta\left[\gamma(\alpha(\overline{r} - c_H) + \beta\lambda(1-\beta)(c_H - c_L)) + (1-\gamma)(\alpha(1-\alpha)(\overline{r} - c_H))\right]$$

from which

$$\frac{\partial \pi}{\partial p_A} = \frac{\tau + p_B - p_A}{2\tau} - p_A \cdot \frac{1}{2\tau} - \frac{\eta}{2\tau}\left(\alpha(\overline{r} - c_H) + \beta\lambda(1-\beta)(c_H - c_L) - \alpha(1-\alpha)(\overline{r} - c_H)\right)$$

[31] While in the proof π denotes profit from a unit customer for a given type, here it means total profit. This slight inconsistency is due to clarity of the respective proofs.

We obtain the symmetric price equilibrium $p^\star$ from the first order condition $\frac{\partial \pi}{\partial p_A} = 0$ and imposing $p_A^\star = p_B^\star$. This leads to:

$$p^\star = \tau - \eta \left[\alpha^2(\overline{r} - c_H) + \lambda\beta(1-\beta)(c_H - c_L)\right]$$

Case 2: $\alpha \geq \beta\lambda$: The profit functions from above:

$$\pi_A = p_A\gamma + \gamma(\alpha(\overline{r} - c_H) + \alpha(1-\beta)(c_H - c_L)) + (1-\gamma)\left((1-\alpha)\alpha(\overline{r} - c_H) + (1-\alpha)(\alpha - \beta\lambda)\frac{1-\beta}{1-\beta\lambda}(c_H - c_L)\right)$$

from which

$$\begin{aligned}\frac{\partial \pi}{\partial p_A} = \frac{\tau + p_B - p_A}{2\tau} - p_A \cdot \frac{1}{2\tau} - \frac{\eta}{2\tau}(&\alpha(\overline{r} - c_H) + \alpha(1-\beta)(c_H - c_L) \\ &- \alpha(1-\alpha)(\overline{r} - c_H) - (1-\alpha)(\alpha - \beta\lambda)\frac{1-\beta}{1-\beta\lambda}(c_H - c_L))\end{aligned}$$

After some algebra:

$$\frac{\partial \pi}{\partial p_A} = \frac{\tau + p_B - p_A}{2\tau} - p_A \cdot \frac{1}{2\tau} - \frac{\eta}{2\tau}(\alpha^2(\overline{r} - c_H) + (\alpha(\alpha - \beta\lambda) + \beta\lambda(1-\alpha))\,\frac{1-\beta}{1-\beta\lambda}\,(c_H - c_L))$$

from which after solving $\frac{\partial \pi}{\partial p_A} = 0$ and imposing $p_A^\star = p_B^\star$:

$$p^\star = \tau - \eta \left[\alpha^2(\overline{r} - c_H) + (\alpha(\alpha - \beta\lambda) + \beta\lambda(1-\alpha))\,\frac{1-\beta}{1-\beta\lambda}(c_H - c_L)\right]$$

From this it's obvious the continuity of PCA prices at the regime change $\alpha = \beta\lambda$. Notice that in a symmetric equilibrium $p^\star$ also represents industry profit from the first (PCA-) stage of bank business.
Derivatives: The derivatives of the (positive) $p^\star$ are as follows:

$$\frac{\partial p}{\partial \alpha} := \begin{cases} -2\eta\alpha(\overline{r} - c_H) < 0 & \text{if } \alpha < \beta\lambda \\ -2\eta\left(\alpha(\overline{r} - c_H) + \frac{\alpha - \beta\lambda}{1-\beta\lambda}(1-\beta)(c_H - c_L)\right) < 0 & \text{otherwise.} \end{cases}$$

$$\frac{\partial p}{\partial \lambda} := \begin{cases} -\eta\beta(1 - \beta(c_H - c_L)) < 0 & \text{if } \alpha < \beta\lambda \\ -\eta\frac{(1-\alpha)^2}{(1-\beta\lambda)^2}\beta(1-\beta)(c_H - c_L) < 0 & \text{otherwise.} \end{cases}$$

Appendix B.3 Proof of Proposition 5.

For part 2 observe that without naivety ($\alpha = 0$) the upper bound of the outsider's pricing distribution is $\hat{r}$. While (B.21) established that this upper bound is given by c_h.

We decompose the outsider's profit function from the proof of Proposition 4 as follows (the proportionality $[\eta\ell_{-j}]$ omitted):

$$\pi^h_{out} = \begin{cases} (\alpha(\beta\lambda(1-\log[\beta\lambda]) - \alpha)(\bar{r} - c_h) - \beta\lambda(c_h - c_l)(1 - \beta\lambda(1 - \log[\beta\lambda]))) & \text{if} \alpha < \beta\lambda \\ (\beta\lambda((\alpha-1)(c_h - c_l) + \alpha\log(\alpha)(c_l - \bar{r}))) & \text{otherwise} \end{cases}$$

and

$$\pi^l_{out} = \begin{cases} (1 - \beta\lambda(1 - \log[\beta\lambda]))\,(\beta\lambda(c_h - c_l) + \alpha(\bar{r} - c_h)) & \text{if} \alpha < \beta\lambda \\ \alpha\,(1 - \alpha + \beta\lambda\log[\alpha]))\,(\bar{r} - c_l) & \text{otherwise} \end{cases}$$

It is trivial to see that substituting $\alpha = 0$ we obtain $\pi^h_{out} = -\pi^l_{in} < 0$, proving statement (2) of the Proposition.

For statement (3), observe that at $\alpha = 1$ the profit $\pi^h_{out} = 0$, but it's derivative w.r.t. α is negative at $\alpha = 1$, which implies that at some $\alpha < 1$ it is positive. Due to continuity, together with the previous claim, the existence of α.

Appendix B.4 Proof of Proposition 6

We prove the Proposition by showing that the respective cumulative distributions of credit fees for each type decrease in α - which means that a price dispersion with higher α first-order stochastically dominates (FOSD) the one with lower α. FOSD trivially implies that expected credit fees are increasing in α.

Naive customers. Naive customers always pay according to the insiders' price dispersion in Proposition 3. We compute

$$\frac{\partial F^\ell_{in}}{\partial\alpha} = \begin{cases} \frac{-1}{1-\beta\lambda}\frac{\bar{r}-c_h}{r-c_\ell} < 0 & \text{if } \alpha < \beta\lambda \\ \frac{-1}{1-\beta\lambda}\frac{\bar{r}-c_\ell}{r-c_\ell} < 0 & \text{otherwise} \end{cases}$$

and

$$\frac{\partial F^h_{in}}{\partial\alpha} = \begin{cases} -\frac{\bar{r}-c_h}{\beta\lambda(r-c_h)} < 1 & \text{if } \alpha < \beta\lambda \\ 0 & \text{otherwise} \end{cases}$$

This implies that for naive customers the expected credit price decreases in α except for naive-high-signal types, for whom the credit price is constant for $\alpha > \beta\lambda$.

Sophisticated customers. The above argument can be applied in a straightforward way to the case $\alpha > \beta\lambda$, as in addition to the above, F_{out} is also FOSD decreases in α. Concretely, for $\alpha > \beta\lambda$ the upper part of the distribution becomes degenerate, and the derivative of the lower part (for $r < \hat{r}$) becomes

$$\frac{\partial F_{out}}{\partial \alpha}_{|\alpha > \beta\lambda} = \frac{-1}{(1-\alpha)^2} \frac{\bar{r} - c_\ell}{r - c_\ell} < 0$$

However, for $\alpha < \beta\lambda$, the distribution is not decreasing in α for the entire domain of $r \in [\underline{r}, \hat{r}]$. Let us define $r_{min} = \min\{r_{in}, r_{out}\}$. Notice that with r_{in} and r_{out} independent random variables one can write the distribution of r_{min} as

$$F_{min}(r) = F_{in}(r) + F_{out}(r) - F_{in}(r) F_{out}(r) \tag{B.36}$$

Region $r < \hat{r}$. Substituting the CDF's from Proposition 3 we obtain:

$$F^\ell_{min} = \frac{1 - \alpha - \beta\lambda - \frac{(\alpha(\bar{r} - c_l) + (\beta\lambda - \alpha)(c_h - c_l))^2}{(r - c_l)^2} + \frac{(\alpha + \beta\lambda)(\alpha(\bar{r} - c_h) + \beta\lambda(c_h - c_l))}{r - c_l}}{(1-\alpha)(1-\beta\lambda)} \tag{B.37}$$

$$\frac{\partial F^\ell_{min}}{\partial \alpha} = -\frac{\beta\lambda \left((\bar{r} - c_h)(2c_h - c_\ell - r) - r(1 - \beta\lambda)(c_\ell - c_h) + c_h(\beta\lambda(c_h - c_\ell) + c_l - 2r) + r^2 \right) + \left(2\alpha - \alpha^2\right)(c_h - \bar{r})(-\bar{r} + c_h - c_\ell + r)}{(1-\alpha)^2 (1 - \beta\lambda)(r - c_\ell)^2}$$

one only need to show positivity of the numerator. The numerator is decreasing in r[32], so one only need to show that it is positive at $\hat{r}$. We obtain

$$\frac{\partial F^\ell_{min}}{\partial \alpha}(\hat{r}) = \frac{(1-\alpha)(\beta\lambda - \alpha)(\beta\lambda(c_h - c_\ell) + \alpha(\bar{r} - c_h))(\bar{r} - c_h)}{\beta\lambda} > 0 \tag{B.38}$$

This concludes the proof that for ℓ-signal type sophisticated customers the credit price always increases in α.
Region $r > \hat{r}$ – with similar arguments one obtains:

$$\frac{\partial F^h_{min}}{\partial \alpha} = -\frac{\alpha(2-\alpha)(\bar{r} - r)}{(1-\alpha)^2 \beta\lambda (r - c_h)^2} \leq 0 \tag{B.39}$$

This concludes the proof that credit price decreases for h-type sophisticated customers as well.

[32] To show this: take the derivative $\frac{\partial^2 F_{min}}{\partial\alpha\partial r}$. This is increasing in r. Substitute $r = \hat{r}$. Show that this is still negative. Details are ommitted for brevity.

Appendix B.5 Proof of Proposition 7

Industry profit in equilibrium, using the proof of Proposition 4:

(1) from credit markets:

$$\pi_{cre} := \begin{cases} \eta\left[\alpha(2-\alpha)(\bar{r}-c_h)+\beta\lambda(1-\beta\lambda)(c_h-c_\ell)\right] & \text{if } \alpha<\beta\lambda \\ \eta\left[\alpha(2-\alpha)(\bar{r}-c_\ell)-\beta\lambda(c_h-c_\ell)\right] \end{cases} \tag{B.40}$$

(2) from PCA markets, if $p \neq 0$ using the substitution $c_h - c_\ell = \frac{1-\beta}{1-\beta\lambda}(c_H - c_L)$:

$$\pi_{pca} = p^\star = \begin{cases} \tau-\eta\left[\alpha^2(\bar{r}-c_h)+\beta\lambda(1-\beta\lambda)(c_h-c_\ell)\right] & \text{if } \alpha<\beta\lambda \\ \tau-\eta\left[\alpha^2(\bar{r}-c_h)+\left[\alpha(\alpha-\beta\lambda)+\beta\lambda(1-\alpha)\right](c_h-c_\ell)\right] & \text{otherwise.} \end{cases} \tag{B.41}$$

(=) aggregate profit, $\pi := \pi_{pca} + \pi_{cre}$

$$\pi = \begin{cases} \tau+2\eta\left[\alpha(1-\alpha)(\bar{r}-c_h)\right] & \text{if } \alpha<\beta\lambda \\ \tau+2\eta\left[(1-\alpha)\left(\alpha(\bar{r}-c_h)+(\alpha-\beta\lambda)(c_h-c_\ell)\right)\right] & \text{otherwise.} \end{cases} \tag{B.42}$$

Notice that the aggreagte profit can be written as

$$\pi := \tau + 2\pi^T_{out}$$

where π^T_{out} is industry profit from banks' role as outsider (from B.34, after substituting $l = l_A + l_B = 1$.)

We compute the derivative w.r.t. α and find the optimum solution:

$$\frac{\partial\pi}{\partial\alpha} = \begin{cases} (2(1-2\alpha))(\bar{r}-c_h) \Rightarrow \alpha^\star = 1/2 \\ 2\left((1-2\alpha)(\bar{r}-c_h)+(1-2\alpha+\beta\lambda)(c_h-c_\ell)\right) \Rightarrow \alpha^\star = \frac{1}{2}+\frac{\beta\lambda(c_h-c_l)}{\bar{r}-c_\ell} \end{cases}$$

The bank profit has a (local) maximum at $\alpha^\star$.[33] Piecewise concavity follows immediately

[33] Precisely, when $\alpha < \beta\lambda$, the profit has a maximum either at $\alpha = 1/2$ or $\alpha = \beta\lambda$ in case $\beta\lambda < 1/2$.

from the second derivatives, which implies customer surplus is piecewise convex.

$$\frac{\partial^2 \pi}{\partial \alpha^2} = \begin{cases} -4(\bar{r} - c_h) < 0 & \text{if } \alpha < \beta\lambda \\ -4(\bar{r} - c_h) - 2(c_h - c_l) < 0 & \text{otherwise.} \end{cases}$$

Appendix B.6 Proof of Proposition 8

The statements for the free-banking case follow from analyzing the credit market profits in Equation (B.40). Notice that c_ℓ is a function of λ. So we rewrite first as

$$\pi_{cre} := \begin{cases} \eta\left[\alpha(2-\alpha)(\bar{r} - c_H) + \beta\lambda(1-\beta)(c_H - c_L\right] & \text{if } \alpha < \beta\lambda \\ \eta\left[\alpha(2-\alpha)(\bar{r} - c_H) - (\alpha(2-\alpha) - \beta\lambda)\frac{1-\beta}{1-\beta\lambda}(c_H - c_L)\right] & \text{otherwise.} \end{cases}$$

The derivatives follow:

$$\frac{\partial \pi_{cre}}{\partial \lambda} = \begin{cases} \beta(1-\beta)(c_H - c_L) > 0 & \text{if } \alpha/\beta < \lambda \\ -\beta(1-\beta)(c_H - c_L) \cdot \frac{(1-\alpha)^2}{(1-\beta\lambda)^2} < 0 & \text{otherwise.} \end{cases}$$

This concludes the first part of the proof (free banking).

From (B.42) it is immediate that bank profit / consumer surplus is not a function of λ for $\alpha/\beta < \lambda$. For the case $\alpha > \lambda\beta$ dependence on λ is only on the last term. Again substituting $c_h - c_l = \frac{1-\beta}{1-\beta\lambda}(c_H - c_L)$ we obtain:

$$\frac{\partial \pi}{\partial \lambda} = -\frac{1-\alpha}{(1-\beta\lambda)^2}\beta(1-\beta)(c_H - c_L) < 0$$

This shows that profit *decreases* in λ, and concludes the proof of the second part.

■

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Appendix C Internet Appendix

Appendix C.1 Expected fee of credit.

We compute expected cost of credit separately for the case $\alpha < \beta\lambda$ and $\alpha \geq \beta\lambda$. For this proof $f_{\cdot}^{\cdot}$ denotes the PDF of the corresponding CDF $F_{\cdot}^{\cdot}$.

Case 1: $\alpha < \beta\lambda$**.** From Proposition 3 we can establish that:

$$\int_{\underline{r}}^{\hat{r}} f_i^l(r)dr = 1 \qquad \int_{\underline{r}}^{\hat{r}} f_o(r)dr = \frac{1-\beta\lambda}{1-\alpha}$$
$$\int_{\hat{r}}^{\overline{r}} f_i^h(r)dr = \frac{\beta\lambda-\alpha}{\beta\lambda} \qquad \int_{\hat{r}}^{\overline{r}} f_{out}(r)dr = \frac{\beta\lambda-\alpha}{1-\alpha}$$
$$\int_{\hat{r}}^{\overline{r}}\int_{\hat{r}}^{r_o} f_o(r_o) * f_i^h(r_i)dr_i dr_o = \frac{(\beta\lambda-\alpha)^2}{2\beta\lambda(1-\alpha)} \tag{C.43}$$

Insider wins clients with an h-signal who are sophisticated with probability given by (C.43). Outsider wins with the complementer probability, or alternatively, it can win h-customers three ways: (i) it offers $r_o < \hat{r}$, (ii) both banks randomize over the range $[\hat{r}, \overline{r})$ and it wins, and (iii) outsider randomizes in this range, but insider plays $\overline{r}$.

$$Pr[r_h \leq r_o] = \frac{(\beta\lambda-\alpha)^2}{2\beta\lambda(1-\alpha)} \tag{C.44}$$
$$Pr[r_o < r_h] = \frac{1-\beta\lambda}{1-\alpha} + \frac{\beta\lambda-\alpha}{1-\alpha}\frac{\beta\lambda-\alpha}{\beta\lambda}\frac{1}{2} + \frac{\beta\lambda-\alpha}{1-\alpha}\frac{\alpha}{\beta\lambda} = \frac{\alpha^2+(\beta\lambda)^2-2\beta\lambda}{2\beta\lambda(\alpha-1)} \tag{C.45}$$

For the l-type customers we calculate similarly:

$$Pr[r_o < r_\ell] = \int_{\underline{r}}^{\hat{r}}\int_{\underline{r}}^{r_i} f_o(r_o) * f_i^l(r_i)dr_o dr_i = \frac{1-\beta\lambda}{2(1-\alpha)} \tag{C.46}$$

Equation (C.46) gives the probability that outsider wins. Insider wins in addition whenever outsider charges more than $\hat{r}$, with probability $(1 - F_o(\hat{r}))$. So:

$$Pr[r_l \leq r_o] = \frac{1-\beta\lambda}{2(1-\alpha)} + \frac{\beta\lambda-\alpha}{1-\alpha} = \frac{1+\beta\lambda-2\alpha}{2(1-\alpha)} \tag{C.47}$$

Expected fees (Case 1): The expected credit fees conditional on signal $\{h, \ell\}$ for sophisticated customers can be formulated generally as

$$\mathbb{E}[r|h,S] := Pr[r_o < r_h]\cdot\mathbb{E}[r_o|r_o < r_h] + Pr[r_h \leq r_o]\cdot\mathbb{E}[r_h|r_h < r_o]$$
$$\mathbb{E}[r|\ell,S] := Pr[r_o < r_h]\cdot\mathbb{E}[r_o|r_o < r_h] + Pr[r_h \leq r_o]\cdot\mathbb{E}[r_h|r_h < r_o]$$

where the respective probabilities and expectations may be piecewise-defined. For naive customers the expected fee is the expected value of the insider's strategy (including any potential mass-point), that is,

$$\mathbb{E}[r|h,N] := \int r f_i^h(r)dr$$
$$\mathbb{E}[r|\ell,N] := \int r f_i^\ell(r)dr$$

These are expectations conditioned on the signal. Expectations on the true type $\{L,H\}$ are then computed by considering the potential misclassification of a H-type:

$$\mathbb{E}[r|h,\cdot] = \lambda\mathbb{E}[r|h,\cdot] + (1-\lambda)\mathbb{E}[r|\ell,\cdot]$$

Below we provide some interim results but omit step-by-step calculations for brevity.
Sophisticated customers. We calculate expected fee offered by the insider resp. the outsider conditional on winning the competition.[34] Let f_{io} denote joint distribution function defined as $f_{io}(r_i,r_o) = f_i(r_i)\cdot f_o(r_o)$.

$$\begin{aligned}
\mathbb{E}[r_o|r_o<r_h] =& \frac{1}{Pr[r_o<r_h]} \times \left(\int_{\hat{r}}^{\overline{r}} \int_{\hat{r}}^{r_h} r_o f_{io}(r_h,r_o)dr_o dr_h + \int_{\underline{r}}^{\hat{r}} r f_o(r)dr + \frac{a}{b}\int_{\hat{r}}^{\overline{r}} r f_o(r)dr \right)\\
\mathbb{E}[r_o|r_o<r_\ell] =& \frac{1}{Pr[r_o<r_\ell]} \times \int_{\underline{r}}^{\hat{r}} \int_{\underline{r}}^{r_\ell} r_o f_{io}(r_\ell,r_o)dr_o dr_\ell\\
\mathbb{E}[r_h|r_h\le r_o] =& \frac{1}{Pr[r_h\le r_o]} \times \left(\int_{\hat{r}}^{\overline{r}} \int_{\hat{r}}^{r_o} r_h f_{io}(r_h,r_o)dr_h dr_o \right)\\
\mathbb{E}[r_\ell|r_\ell\le r_o] =& \frac{1}{Pr[r_\ell\le r_o]} \times \left(\int_{\underline{r}}^{\hat{r}} \int_{\underline{r}}^{r_o} r_\ell f_{io}(r_\ell,r_o)dr_\ell dr_o + \hat{\rho}\times \int_{\underline{r}}^{\hat{r}} r f_i^l(r)dr \right)
\end{aligned} \tag{C.48}$$

First, notice that whenever a joint distribution function $f_{xy}(x,y)$ is symmetric in the two variables, the following is true:

$$\int_a^b \int_a^y x f_{xy}(x,y)dxdy = \int_a^b \int_a^x y f_{xy}(x,y)dydx$$

Let $\Phi^{(\ell)}$denote this common value of the double-integrals over $[\underline{r},\hat{r})$ and $\Phi^{(h)}$ over $[\hat{r},\overline{r})$ using the appropriate joint distributions from Proposition 3. We present these below[35]:

$$\begin{aligned}
\Phi^{(\ell)} &= \frac{1}{1-\alpha}\left(\underline{r} - \frac{c_\ell(1+\beta\lambda)}{2} + \frac{\beta\lambda\log(\beta\lambda)}{1-\beta\lambda}(\underline{r}-c_\ell)\right)\\
\Phi^{(h)} &= \frac{\alpha-\beta\lambda}{(1-\alpha)\beta\lambda}\left(\frac{\alpha-\beta\lambda}{2}c_h + \alpha c_h - \alpha\overline{r}\right) - \frac{\alpha^2\log(\beta\lambda/\alpha)}{(1-\alpha)\beta\lambda}(\overline{r}-c_h)
\end{aligned}$$

[34]We apply the formula for conditional expectations, $\mathbb{E}[x|x<y] = \frac{\int\int xf(xy)dxdy}{\int\int f(xy)dxdy}$

[35]Step-by-step calculations omitted for brevity.

Further, to compute analytically all formulas in C.48 we need the following integrals:

$$
\begin{aligned}
\int_{\underline{r}}^{\hat{r}} r f_o(r) dr &= \frac{(1-\beta\lambda)c_\ell - \log(\beta\lambda)\left(\underline{r} - c_\ell\right)}{1-\alpha} \\
\int_{\hat{r}}^{\overline{r}} r f_o(r) dr &= \frac{\alpha\left(\overline{r} - c_h\right)\log\left(\frac{\beta\lambda}{\alpha}\right) + c_h(\beta\lambda - \alpha)}{1-\alpha} \\
\int_{\underline{r}}^{\hat{r}} r f_i^l(r) dr &= c_\ell - \frac{(\beta\lambda(c_h - c_\ell) + \alpha(\overline{r} - c_h))\log[\beta\lambda]}{1-\beta\lambda} \\
\int_{\hat{r}}^{\overline{r}} r f_i^h(r) dr &= c_h - \frac{\alpha c_h + \alpha\log[\alpha/\beta\lambda](\overline{r} - c_h)}{\beta\lambda}
\end{aligned}
\tag{C.49}
$$

To derive credit fees we multiply (C.48) with the respective switching probabilities. Sophisticated customers with signal ℓ and h respectively obtain:

$$
\begin{aligned}
\mathbb{E}[r|h,S] &= c_h + \frac{\beta\lambda - 1 - \beta\lambda\log(\beta\lambda)}{1-\alpha}(c_h - c_\ell) + \frac{\alpha\left(\alpha\log[\frac{\alpha}{\beta\lambda}] - \beta\lambda\log(\beta\lambda) + 2(\beta\lambda - \alpha)\right)}{\beta\lambda(1-\alpha)}(\overline{r} - c_h) \\
\mathbb{E}[r|\ell,S] &= c_\ell + \frac{2\beta\lambda(1-\beta\lambda) + (\alpha+\beta\lambda)\beta\lambda\log(\beta\lambda)}{(1-\alpha)(1-\beta\lambda)}(c_h - c_\ell) + \frac{2\alpha(1-\beta\lambda) + \alpha(\alpha+\beta\lambda)\log(\beta\lambda)}{(1-\alpha)(1-\beta\lambda)}(\overline{r} - c_h)
\end{aligned}
\tag{C.50}
$$

We want to know what the *true* type $\{L, H\}$ obtains rather than the classified type $\{l, h\}$, and instead in terms of c_ℓ, we want to see the structural parameters $\{c_L, c_H\}$. For this, we multiply the above expected fees with the probability of a H-type receiving a signal $\{l, h\}$ and replace c_ℓ. (Note that L-type is always correctly classified.)

$$
\mathbb{E}[r|H,S] = \lambda\mathbb{E}[r|h,S] + (1-\lambda)\mathbb{E}[r|\ell,S] \tag{C.51}
$$

$$
\mathbb{E}[r_L,S] = \mathbb{E}[r|\ell,S] \tag{C.52}
$$

Finally, unconditional on type, sophisticated customers obtain

$$
\mathbb{E}[r]^S = [(1-\beta\lambda)c_\ell + \beta\lambda c_h] + \left[2\alpha + \frac{\alpha^2\log[\alpha]}{1-\alpha}\right](\overline{r} - c_h) + \frac{\beta\lambda(1-\beta\lambda) + \alpha\beta\lambda\log[\beta\lambda]}{1-\alpha}(c_h - c_\ell) \tag{C.53}
$$

Notice that $(1-\beta\lambda)c_\ell + \beta\lambda c_h = (1-\beta)c_L + \beta c_H$, so the first square bracket is the average cost. The second can be interpreted as a markup due to customer naiveté and it parallels the case with naivete only. The last term is the extra correction for adverse selection. Also notice that

$$
c_h - c_\ell = \frac{1-\beta}{1-\beta\lambda}(c_H - c_L)
$$

Naive customers: The ℓ-type pays the expected value of r with respect to the distribution $F_i^\ell(r)$. The h-type naive customers pay according to F_i^h which includes mass point at $\overline{r}$ with probability $a/b\lambda$. Therefore, from (C.49)

$$\begin{aligned}
\mathbb{E}[r|\ell, N] &= c_\ell - \frac{(\beta\lambda(c_h - c_\ell) + \alpha(\overline{r} - c_h))\log[\beta\lambda]}{1-\beta\lambda} \\
\mathbb{E}[r|h, N] &= c_h - \frac{\alpha c_h + (\alpha\log[\alpha/\beta\lambda])(\overline{r} - c_h)}{\beta\lambda} + \frac{\alpha}{\beta\lambda}\overline{r}
\end{aligned}$$

Naive customers with *true* type $\{L, H\}$ pay:

$$\begin{aligned}
\mathbb{E}[r|H, N] &= \lambda\mathbb{E}[r|h, N] + (1-\lambda)\mathbb{E}[r|\ell, N] \\
\mathbb{E}[r|L, N] &= \mathbb{E}[r|\ell, N]
\end{aligned}$$

Unconditionally:

$$\mathbb{E}[r|N] = (1-\beta\lambda)c_L + \beta\lambda c_h + (\alpha - \alpha\log[\alpha])(\overline{r} - c_h) - \beta\lambda\log[\beta\lambda](c_h - c_\ell)$$

Notice that when adverse selection is turned off (for example by setting $\beta\lambda = 1$[36]), we obtain for both $\mathbb{E}[r]_S$ and $\mathbb{E}[r]_N$ the appropriate expressions in Lemma 1.

Case 2: $\alpha > \beta\lambda$. From Proposition 3 in this case F_i^h is degenerate, and h-signal always attracts an insider offer of $r_h = \overline{r}$. This implies the outsider always gets all (sophisticated) h-type customers no matter its offer $r < \overline{r}$. Switching probabilities are:

$$Pr[r_o < r_\ell] = \int_{\underline{r}}^{\overline{r}}\int_{\underline{r}}^{r_i} f_{io}(r_i, r_o)dr_o dr_i + \overline{\rho}_{il} = \frac{1-\alpha}{2(1-\beta\lambda)} + \frac{\alpha-\beta\lambda}{1-\beta\lambda} = \frac{1+\alpha-2\beta\lambda}{2(1-\beta\lambda)}$$

The complementary probability is:

$$Pr[r_\ell \leq r_o] = \frac{1-\alpha}{2(1-\beta\lambda)}$$

Expected fees conditional on switching or staying (for the l-type) are derived from the standard formulas:

$$\begin{aligned}
\mathbb{E}[r_o|r_o < r_\ell] &= \frac{1}{Pr[r_o < r_\ell]} \times \left(\int_{\underline{r}}^{\overline{r}}\int_{\underline{r}}^{r_i} r_o f_{io}(r)dr_o dr_\ell + Pr[r_l = \overline{r}] \cdot \int_{\underline{r}}^{\overline{r}} r_o f_o(r_o)d_o\right) \\
\mathbb{E}[r_\ell|r_\ell \leq r_o] &= \frac{1}{Pr[r_\ell \leq r_o]} \times \left(\int_{\underline{r}}^{\overline{r}}\int_{\underline{r}}^{r_o} r_\ell f_{io}(r)dr_\ell dr_o\right)
\end{aligned}$$

The double integrals are:

$$\Phi := \frac{\alpha(\alpha\log[\alpha] + 1 - \alpha)}{(1-\alpha)(1-\beta\lambda)}(\overline{r} - c_\ell) + \frac{1-\alpha}{2(1-\beta\lambda)}c_\ell$$

[36] Calculating $\beta\lambda = 0$ is not possible due to our assumption $\beta\lambda > \alpha$.

Expected fees for sophisticated l-types are:

$$\mathbb{E}[r|l,S] = Pr[r_o < r_\ell] \times \mathbb{E}[r_o|r_o < r_\ell] + Pr[r_\ell < r_o] \times \mathbb{E}[r_\ell|r_\ell < r_o] = 2\Phi + \frac{\alpha - \beta\lambda}{1-\beta\lambda} \cdot \int_{\underline{r}}^{\overline{r}} r_o f_o(r_o) d_o,$$

while the sophisticated h-types, who always switch, pay

$$\mathbb{E}[r|h,S] = \int_{\underline{r}}^{\overline{r}} r f_o dr = c_\ell - \frac{\alpha \log[\alpha]}{1-\alpha}(\overline{r} - c_\ell)$$

ℓ-type simplifies to:

$$\mathbb{E}[r|\ell,S] = c_\ell + \frac{\alpha(2 - 2\alpha + \log[\alpha](\alpha + \beta\lambda))}{(1-\alpha)(1-\beta\lambda)}(\overline{r} - c_l) \tag{C.54}$$

The sophisticated *true* ($\{L, H\}$)-types pay the following in expectation:

$$\mathbb{E}[r|H,S] = Pr[h|H]\mathbb{E}[r|h] + Pr[l|H]\mathbb{E}[r|l] = \lambda\left(c_\ell - \frac{\alpha(\overline{r} - c_\ell)\log[\alpha]}{1-\alpha}\right) + (1-\lambda)\mathbb{E}[r|\ell,S]$$
$$\mathbb{E}[r|L,S] = \mathbb{E}[r|\ell]$$

Unconditionally, the sophisticated customers pay:

$$\mathbb{E}[r|S] = (1-\beta\lambda)\mathbb{E}[r|l,s] + \beta\lambda\mathbb{E}[r|h,s] = c_l + \frac{\alpha(2 - 2\alpha + \alpha\log[\alpha])}{1-\alpha}(\overline{r} - c_l) \tag{C.55}$$

Naive customers conditional on the signal pay:

$$\mathbb{E}[r|h,N] = \overline{r}$$
$$\mathbb{E}[r|l,N] = \overline{\rho}_{il}\overline{r} + \int_{\underline{r}}^{\overline{r}} f_i^l(r)dr = \frac{\alpha - \beta\lambda}{1-\beta\lambda}\overline{r} + \frac{(1-\alpha)c_\ell - \alpha\log[\alpha](\overline{r} - c_\ell)}{1-\beta\lambda}$$

Naive customers with *true* type $\{L, H\}$ pay:

$$\mathbb{E}[r|H] = Pr[h|H]\overline{r} + Pr[l|H]\mathbb{E}[r|l,N] = \lambda\overline{r} + (1-\lambda)\mathbb{E}[r|l,N]$$
$$\mathbb{E}[r|L] = \mathbb{E}[r|l,N]$$

Naive customers unconditional on signal pay:

$$\mathbb{E}[r|N] = (1-\beta\lambda)\mathbb{E}[r|l,N] + \beta\lambda\overline{r} = \alpha\overline{r} + (1-\alpha)c_\ell - \alpha\log[\alpha](\overline{r} - c_\ell) \tag{C.56}$$

For transparency all expected returns from the proof are summarized below.

$$\mathbb{E}[r|S] = \begin{cases} \overline{c} + \left[2\alpha + \frac{\alpha^2 \log[\alpha]}{1-\alpha}\right](\overline{r} - c_h) + \frac{\beta\lambda(1-\beta\lambda)+\alpha\beta\lambda\log[\beta\lambda]}{1-\alpha}(c_h - c_\ell) & \text{if } \alpha < \beta\lambda \\ c_l + \frac{\alpha(2-2\alpha+\alpha\log[\alpha])}{1-\alpha}(\overline{r} - c_l) & \text{otherwise.} \end{cases} \tag{C.57}$$

$$\mathbb{E}[r|N] = \begin{cases} \overline{c} + (\alpha - \alpha\log[\alpha])(\overline{r} - c_h) - \beta\lambda\log[\beta\lambda](c_h - c_\ell) & \text{if } \alpha < \beta\lambda \\ c_\ell + (\alpha - \alpha\log[\alpha])(\overline{r} - c_\ell) & \text{otherwise} \end{cases} \tag{C.58}$$